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## Antenna Radiation Resistance in Waveguide and in Free-Space

Pavel V. Nikitin and Daniel D. Stancil

**Abstract**—In this paper, we discuss the connection between radiation resistance formulas for antenna in free-space and in rectangular waveguide. We demonstrate with the example of monopole antenna that a transformation between the angular direction in free-space and mode indices in waveguide makes the formulas for antenna radiation resistance in free-space and in rectangular waveguide equivalent to each other.

**Index Terms**—Antenna input impedance, antenna theory, waveguide excitation.

### I. INTRODUCTION

Radiation resistance of antennas radiating into free-space has been studied by many researchers [1]. In free-space, expressions can be derived for the contribution to the radiation resistance from the radiation into certain solid angle.

Manuscript received November 22, 2004; revised January 6, 2005.

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Digital Object Identifier 10.1109/TAP.2005.848524

Antennas coupled into a microwave guide (usually operated in one- or two-mode regime) have also been extensively studied [2], [3]. In waveguides, the only direction of field propagation is along the waveguide. An antenna can be characterized by the waveguide mode distribution that it excites and radiation resistance of such antennas can be derived as a function of mode indices.

In free-space any radiation pattern can be approximated with an infinite sum of waves radiated in different directions just as in waveguides the propagating field can always be decomposed into a sum of normal modes. As the waveguide cross-section becomes large, the number of modes increases, approaching a continuous spectrum of free-space. Waveguide propagation in waveguides can be treated using both ray and mode theories, and equivalence of the fields derived from both approaches has been previously shown for the parallel plate waveguide [4].

In this paper, we demonstrate with the example of a monopole antenna that a transformation between the angles and the mode indices connects the formulas for antenna radiation resistances in free-space and in rectangular waveguide. These formulas and connection between them are derived independently from first principles.

To the best of our knowledge, the only existing work on this topic is our conference publication [5], which treated a limited case and unfortunately contained several errors. The present paper supersedes [5]. The application that stimulated our interest in this topic is study of antenna behavior in HVAC ducts [6].

### II. MONOPOLE ANTENNA

One of the simplest antennas is a monopole probe. Fig. 1 shows a monopole antenna positioned above the ground plane in free-space and coupled into a rectangular waveguide. Radiation resistance of this antenna in both cases can be calculated assuming a sinusoidal current distribution, typical on such an antenna [1], [7].

#### A. Radiation Resistance in Free-space

The characteristics of monopole antenna above a ground plane in free-space are well known [1]. The contribution to the radiation resistance resulting from the power radiated into the angular direction  $(\theta, \phi)$  is

$$dR(\theta, \phi) = \eta \frac{1}{4\pi^2 \sin^2 kl} \frac{[\cos(kl \cos \theta) - \cos kl]^2}{\sin \theta} d\theta d\phi \quad (1)$$

where  $l$  is antenna length,  $\eta$  is the free-space impedance, and  $k = \omega/c$  is the wave number.

#### B. Radiation Resistance in Rectangular Waveguide

Radiation resistance due to the power radiated into the mode  $(n, m)$  can be found to be

$$R(n, m) = 4 \frac{p_{nm} |C_{nm}|^2}{I_o^2}, \quad (2)$$

where  $p_{nm}$  is the normalized power carried by mode  $(n, m)$  in one direction (see [8]),  $C_{nm}$  are mode amplitudes excited by the transmitting antenna, and  $I_o$  is current amplitude on the antenna. Coefficients  $C_{nm}$  can be expressed [9] as

$$C_{nm} = -\frac{I_o}{4p_{nm}} \int_1 \vec{\varepsilon}_{nm} \cdot \vec{j} dl \quad (3)$$

where  $\vec{\varepsilon}_{nm}$  is the normalized electric field of mode  $(n, m)$  along the antenna,  $\vec{j}$  is the normalized surface current density distribution, and

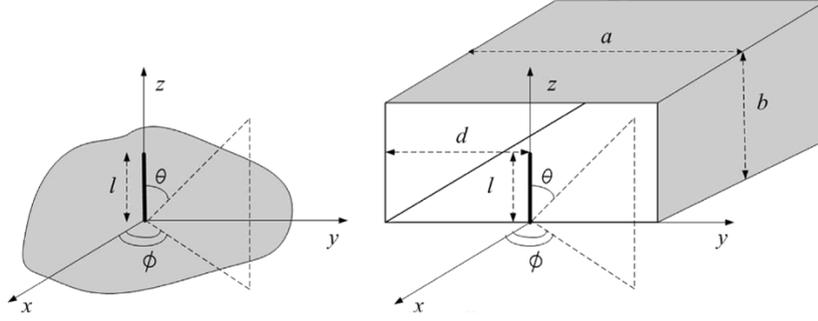
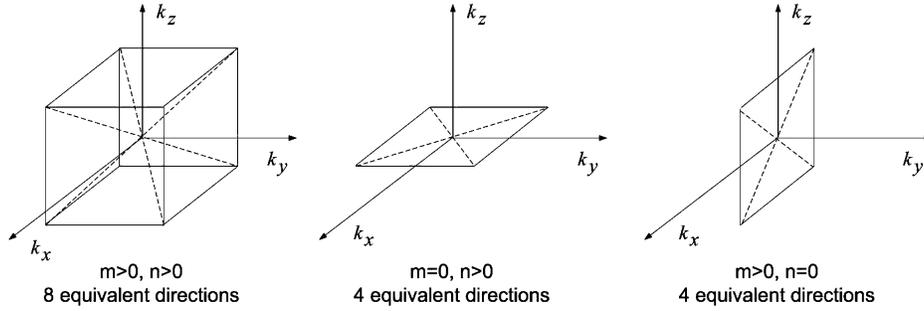


Fig. 1. Monopole antenna in free-space and in rectangular waveguide.


 Fig. 2. Equivalent directions  $N_D$ .

the integral is taken along the antenna length. Contribution to the radiation resistance from  $(n, m)$  modes can be calculated to be

$$R(n, m) = \eta \frac{\chi_n \chi_m \pi^2 \sin^2 \frac{n\pi d}{a} (\cos \frac{m\pi l}{b} - \cos kl)^2}{2k^2 ab g^2 \sin^2 kl} \frac{1}{\left[1 - \left(\frac{m\pi}{kb}\right)^2\right]^2} \times \begin{cases} \frac{kn^2}{\beta a^2} & \text{for } TE \text{ modes} \\ \frac{\beta m^2}{kb^2} & \text{for } TM \text{ modes} \end{cases} \quad (4)$$

where  $\beta = \sqrt{k^2 - g^2}$  is the waveguide wave number of the mode,  $g = \sqrt{((n\pi)/a)^2 + ((m\pi)/b)^2}$  is the cutoff wave number, and  $\chi_l = 1 + \text{sign}(l)$ . By summing contributions from  $TE$  and  $TM$  modes, one can show that

$$R(n, m) = \eta \frac{\chi_n \chi_m \sin^2 \frac{n\pi d}{a}}{2k^2 ab \sin^2 kl} \times \frac{(\cos \frac{m\pi l}{b} - \cos kl)^2}{\left[1 - \left(\frac{m\pi}{kb}\right)^2\right] \sqrt{1 - \left(\frac{n\pi}{ka}\right)^2 - \left(\frac{m\pi}{kb}\right)^2}}. \quad (5)$$

### III. THE TRANSFORMATION

Consider the following transformation, independent of antenna type:

$$\cos \theta = \frac{m\pi}{kb}, \quad \sin \theta \sin \phi = \frac{n\pi}{ka}. \quad (6)$$

The above formula is known [10] and essentially gives the angles of incidence of the mode wavefront normal in a rectangular waveguide. The double differential  $d\theta d\phi$  can be expressed via the Jacobian as

$$\begin{aligned} d\theta d\phi &= \left| \frac{\partial(\theta, \phi)}{\partial(m, n)} \right| dm dn \\ &= \frac{\pi^2}{k^2 ab} \frac{dm}{\sqrt{1 - \left(\frac{m\pi}{kb}\right)^2}} \frac{dn}{\sqrt{1 - \left(\frac{n\pi}{ka}\right)^2 - \left(\frac{m\pi}{kb}\right)^2}}. \end{aligned} \quad (7)$$

### IV. THE CONNECTION

If we restrict  $m, n$  to positive values (as in mode nomenclature), then there are  $N_D$  directions that correspond to coupling with waveguide modes with indices  $m, n$  as shown in Fig. 2. This can also be expressed mathematically as  $N_D = 2\chi_n \chi_m$ .

Using the transformation described in the previous section, the formula for radiation resistance of monopole in free-space given by (1) turns into

$$\begin{aligned} dR(n, m) &= \eta \frac{N_D}{4k^2 ab \sin^2 kl} \\ &\times \frac{(\cos \frac{m\pi l}{b} - \cos kl)^2 dm dn}{\left[1 - \left(\frac{m\pi}{kb}\right)^2\right] \sqrt{1 - \left(\frac{n\pi}{ka}\right)^2 - \left(\frac{m\pi}{kb}\right)^2}}. \end{aligned} \quad (8)$$

Now look at the formula for radiation resistance in rectangular waveguide given by (5). If we take  $d = a/2$ , by symmetry, only modes with odd  $n$  will be excited. If we further note that  $dn = dm = 1$  and  $N_D = 2\chi_n \chi_m$ , then the two radiation resistance formulas given by (5) and (8) become identical.

### V. DISCUSSION

For illustrative purpose, we considered only a monopole antenna. The transformation we used holds for other types of antennas as well but is limited to rectangular waveguides and antennas symmetrically located in them. Circular waveguides and nonsymmetrically located antennas require further investigation.

A possible application of the demonstrated connection is the calculation of the radiation resistance for any antenna in a rectangular waveguide if free-space radiation resistance of the antenna is known (and vice versa). This can be used for analysis of antennas used for HVAC duct communications, such as various monopoles. Key steps in performing the calculation are quantization of transverse components of wavevector, accounting for all space directions coupling to the same mode, and using symmetry arguments to determine modes that are not excited.

## VI. CONCLUSION

In this paper, we have discussed the theoretical equivalence between the radiation resistances of antenna in free-space and in rectangular waveguide. We have demonstrated with the example of monopole antenna that they can be transformed one into another via a transformation between the angles of incidence and the mode indices.

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