Impedance of a Short Dipole Antenna in a Cold Plasma

Pavel Nikitin and Charles Swenson

Abstract—This paper presents the analysis of the impedance of a short dipole antenna in a cold plasma using a quasi-static approach. Two radically different current distributions on the antenna are considered and their influences on the antenna impedance are studied. These distributions include a one-dimensional (1-D) triangular distribution and a three-dimensional (3-D) exponential current distribution. Balmain analytically solved the first problem. Staras proposed the second one. In this paper, we offer an analytical solution for the second one as proposed by Staras. We compare both distributions and find that impedances resulting from them are remarkably close. We conclude that the impedance of a short antenna, as derived using the quasi-static approximation, is insensitive to the choice of current distribution. Therefore, any of these two theories can be used when analyzing data from an impedance probe immersed in a space plasma.

Index Terms—Dipole antennas, impedance, plasma covered antennas, probe antennas.

I. INTRODUCTION

THE ELECTRICAL impedance of an antenna exposed to the space environment depends upon the parameters of the space plasma in which it is immersed (electron density, collision frequencies, external magnetic field, temperature). This effect was first observed by Jackson and Kane [1]. Since then, a number of probes have been built and flown in order to measure parameters of the space environment, based upon this effect [2]–[4]. Recent developments in ionospheric probe technology allow for fast frequency sweeps and for recording of both magnitude and phase of the antenna impedance. Proper analysis of data from such probes requires an accurate theory for the probe impedance. In analyzing the impedance of an antenna immersed in a plasma, all of the following simplifying assumptions listed below or some combination of them are typically made in order to make this a tractable problem. The current distribution on the surface of the antenna cannot be measured directly and thus, is assumed. The quasi-static approach [5], which accounts accurately for the fields in the vicinity of a short dipole, is typically used. The dielectric tensor is usually derived using the fluid model for a cold plasma.

Balmain [6] used a quasi-static cold plasma theory to derive an expression for the impedance of a cylindrical dipole antenna. In his work, the triangular current distribution was con-

sidered as the near-field source allowing him to obtain his well-known simple analytical expressions, which he later refined [7]. Staras [8] proposed a cylindrically symmetrical exponentially decaying current (discussed later as a three-dimensional (3-D) exponential current), which is attractive because of its simple analytical properties. However, Staras was not able to derive a solution for this problem.

Lafon and Weil [9] investigated what convergence properties an arbitrary current distribution must satisfy in order for impedance integrals, computed in a full-wave case, to converge. Other researchers approached this problem mostly using a surface triangular current distribution and a few other simplifying assumptions, including either a collisionless model for a plasma [10] or a transmission line approach to computing the impedance [11].

Balmain [6] showed that the impedances resulting from two different one-dimensional (1-D) current distributions are very similar. In this paper, we will complete Staras' work by finding a closed-form solution for an impedance of an antenna with a 3-D exponential current aligned with the external magnetic field. We will show that this impedance is very close to the one obtained by Balmain.

II. THEORY

We will briefly outline the quasi-static method for calculating the impedance of a dipole antenna immersed in a plasma, as used by a number of previously-mentioned authors. This method does not include electromagnetic (radiation) effects, which have very little effect on the reactance of short antennas in a plasma [6], [12]. The antenna resistance, predicted by this method, is due only to plasma losses. The geometry of the problem is shown in Fig. 1.

When the current density on the surface of the antenna is known, the antenna impedance can be derived as an integral of the near-zone electric field. Assume the following time dependence for the current density $\vec{J}(\vec{r}, t)$ and electric field $\vec{E}(\vec{r}, t)$:

$$\vec{J}(\vec{r}, t) = \vec{J}(\vec{r})e^{j\omega t} \tag{1}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_\omega(\vec{r})e^{j\omega t} \tag{2}$$

where $\vec{J}(\vec{r})$ is the current distribution on the antenna and $\vec{E}_\omega(\vec{r})$ is the electric field distribution at the driving frequency ω. The impedance integral can be written as

$$Z = -\frac{1}{I_0^2} \int_A \vec{J}(\vec{r})\vec{E}_\omega(\vec{r})\,d\vec{r} \tag{3}$$
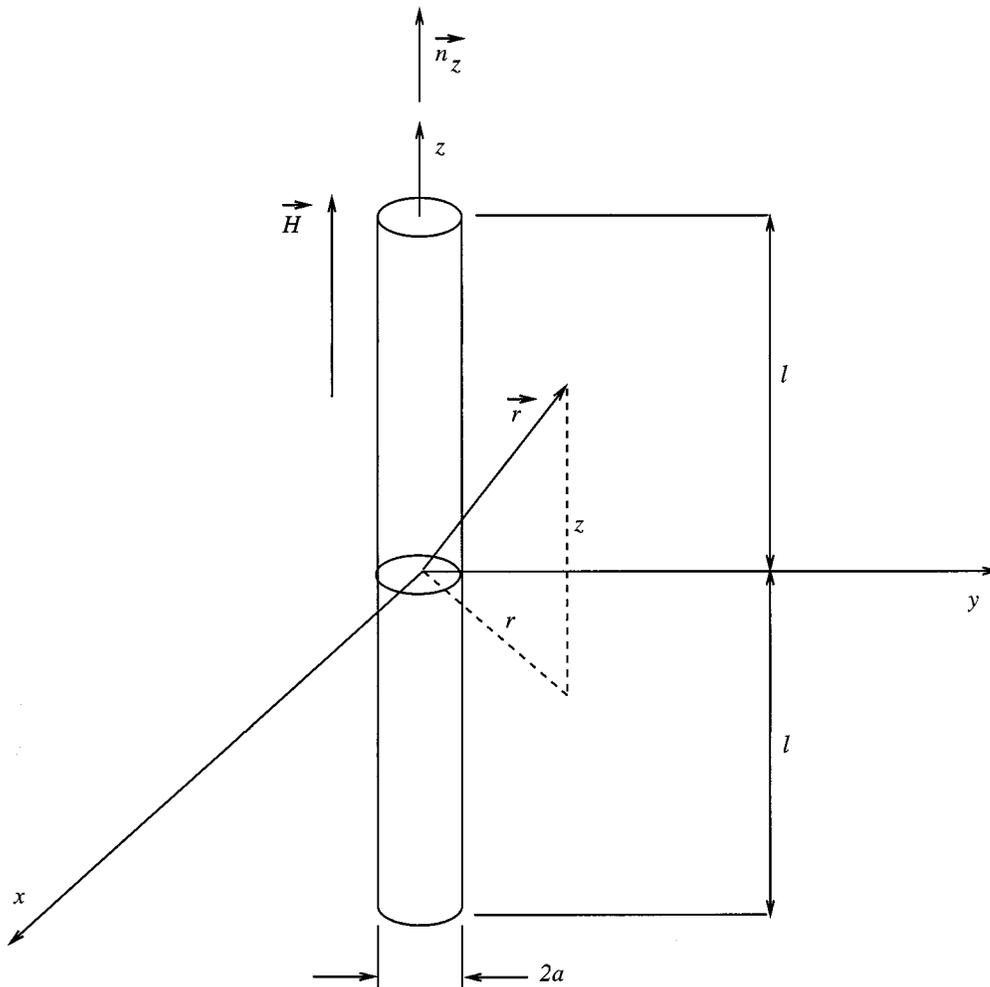
Fig. 1.   Dipole antenna oriented along the magnetic field.

where I_0 is the magnitude of the current at the antenna terminals and the integral is taken over the surface A of the antenna. Using the $\vec{k}$-space Fourier transform defined symmetrically as

$$\mathcal{F}(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int_{\vec{r}} f(\vec{r}) e^{j\vec{k}\vec{r}} \, d\vec{r} \qquad (4)$$

allows us to simplify the computation of an integral and to rewrite (3) as

$$Z = -\frac{1}{I_0^2} \int_{\vec{k}} \vec{\mathcal{J}}(\vec{k}) \vec{\mathcal{E}}_\omega(\vec{k}) \, d\vec{k} \qquad (5)$$

where the integral is now taken over the whole $\vec{k}$-space and $\vec{\mathcal{J}}$ and $\vec{\mathcal{E}}_\omega$ are the $\vec{k}$-space Fourier transforms of $\vec{J}$ and $\vec{E}_\omega$, respectively.

The plasma can be approximated as a dielectric medium with permittivity tensor $\hat{\varepsilon}$. For a cold plasma with the magnetic field along the z-axis, this tensor, as derived from a fluid model is

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \qquad (6)$$

where

$$\varepsilon_1 = 1 - \frac{\omega_p^2 \left(1 - j\frac{\nu}{\omega}\right)}{\omega^2 \left(1 - j\frac{\nu}{\omega}\right)^2 - \omega_c^2}$$

$$\varepsilon_2 = \frac{\omega_p^2 \omega_c}{\omega(\omega^2 - \omega_c^2)}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2 \left(1 - j\frac{\nu}{\omega}\right)} \qquad (7)$$

and where ω_p, ω_c, ν are the plasma frequency, the cyclotron frequency, and the collision frequency, respectively. The quasi-static approach implies that the electric field can be expressed purely in terms of scalar potential. The electric field $\vec{\mathcal{E}}_\omega$ can be found as a function of $\vec{\mathcal{J}}$ after inverting Maxwell's equations

$$\vec{\mathcal{E}}_\omega = -\frac{j\vec{k}\left[\vec{\mathcal{J}} \cdot \vec{k}\right]}{\omega\varepsilon_0 \left[\vec{k} \cdot \hat{\varepsilon} \cdot \vec{k}\right]}. \qquad (8)$$
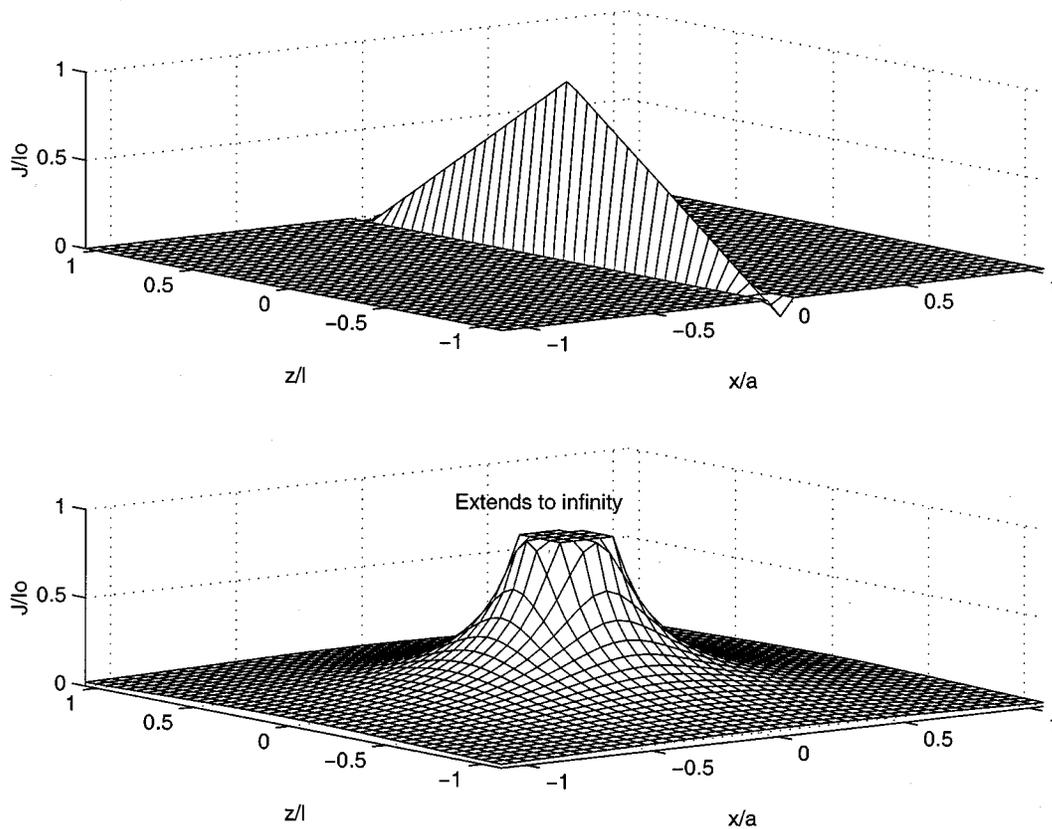
Fig. 2. Normalized current amplitude for (a) 1-D triangular and (b) 3-D exponential current distributions in the x, z plane at $y = 0$.

The expression for impedance in cylindrical coordinates then becomes

$$Z = -\frac{j}{\omega \varepsilon_0 I_0^2} \int_{\vec{k}} \frac{(\vec{\mathcal{J}} \cdot \vec{k})^2}{\varepsilon_1 k_r^2 + \varepsilon_3 k_z^2} \, d\vec{k}. \tag{9}$$

A. Current Distributions

In this section we will describe two current distributions for a cylindrical dipole antenna of radius a and length $2l$. These distributions are not physical but they allow us to derive an analytical solution for antenna impedance. Current amplitudes for both distributions are shown in Fig. 2 in the x, z plane at $y = 0$.

The 1-D triangular current distribution used by Balmain assumes that an antenna is infinitesimally thin. The current and its Fourier transform for this case are

$$\vec{J}(\vec{r}) = I_0 \left(1 - \frac{|z|}{l}\right) \frac{\delta(x)\,\delta(y)}{2\pi a} \vec{n}_z \tag{10}$$

$$\vec{\mathcal{J}}(\vec{k}) = \frac{l I_0}{(2\pi)^{3/2}} \operatorname{sinc}^2 \left(\frac{k_z l}{2}\right) \vec{n}_z. \tag{11}$$

The 3-D exponential current distribution proposed by Staras [8] is not confined to the surface of the antenna, the current decays

away exponentially with radial distance from the surface. The current and its Fourier transform for this case are

$$\vec{J}(\vec{r}) = \frac{I_0 e^{-\sqrt{\frac{r^2}{a^2} + \frac{z^2}{l^2}}}}{2\pi a^2 \sqrt{\frac{r^2}{a^2} + \frac{z^2}{l^2}}} \vec{n}_z \tag{12}$$

$$\vec{\mathcal{J}}(\vec{k}) = \frac{2l I_0}{(2\pi)^{3/2}(1 + k_r^2 a^2 + k_z^2 l^2)} \vec{n}_z. \tag{13}$$

B. Antenna Impedance

Balmain [6] published an analytical solution for the impedance of a dipole antenna immersed in a cold collisional plasma assuming a 1-D triangular current distribution For the cylinder axis aligned with the magnetic field, this solution is

$$Z = -\frac{j}{2\pi \omega \varepsilon_0 \varepsilon_1 l} \left[\ln \frac{l}{a} - 1 + \ln \left(\frac{\varepsilon_1}{\varepsilon_3}\right)^{1/2} \right]. \tag{14}$$

To find a solution for the same antenna with the 3-D exponential current distribution, proposed by Staras, we need to solve the following impedance integral:

$$Z = -\frac{j}{\pi^2 \omega \varepsilon_0 \varepsilon_3 l} \int_0^{+\infty} \int_{-\infty}^{+\infty}$$

$$\cdot \frac{s^2 t \, ds \, dt}{\left(s^2 + \frac{\varepsilon_1}{\varepsilon_3} t^2\right) \left(s^2 + 1 + \frac{a^2}{l^2} t^2\right)^2} \tag{15}$$
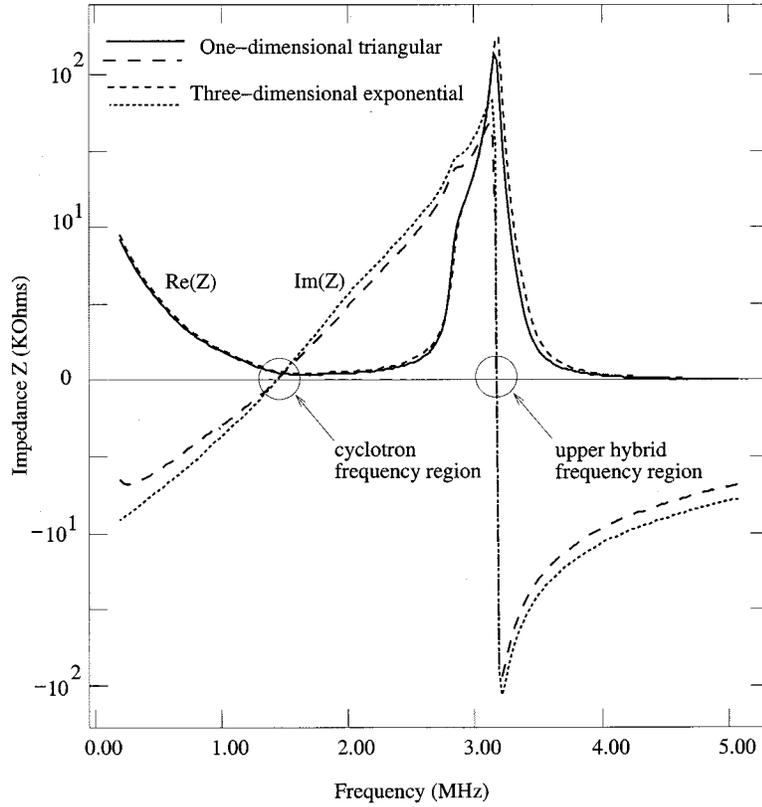
Fig. 3. Comparison of impedances of antennas with 1-D triangular and 3-D exponential current distributions for a typical Earth's ionosphere. $l = 0.5$ m, $a = 0.01$ m, $f_c = 1.44$ MHz, $f_p = 2.84$ MHz, $f_\nu = 0.05$ MHz.

where $s = k_z l$ and $t = k_r l$ are unitless variables. The inner integral can be computed from Cauchy's theorem

$$G(p, q) = \int_{-\infty}^{+\infty} \frac{s^2 \, ds}{(s^2 - q^2)(s^2 + p^2)^2} = \frac{\pi}{2p(p - jq)^2} \quad (16)$$

where $q = t\sqrt{-\varepsilon_1/\varepsilon_3}$ and $p^2 = 1 + a^2 t^2/l^2$. Because the plasma is collisional, the poles $\pm q$ do not lie on the real axis and the complex contour is closed in the upper half-plane.

We can write the impedance integral as

$$Z = -\frac{j}{2\pi\omega\varepsilon_0\varepsilon_3 l} \int_0^{+\infty} \frac{t \, dt}{p(p - jq)^2} = \text{Re}(Z) + j \, \text{Im}(Z). \quad (17)$$

Substituting values for p, q, ε_3 and performing a change of variables $1/(2y) - y/2 = at/l$ we can define the impedance in terms of single-quadrature expressions

$$\text{Re}(Z) = \frac{1}{2\pi\omega\varepsilon_0 l} \frac{\left[\frac{\nu\omega_p^2}{\omega^3} \mathcal{I}_1 + \left(1 - \frac{\omega_p^2 - \nu^2}{\omega^2}\right)\mathcal{I}_2\right]}{\left[\left(1 - \frac{w_p^2}{\omega^2}\right)^2 + \frac{\nu^2}{\omega^2}\right]} \quad (18)$$

and

$$\text{Im}(Z) = \frac{1}{2\pi\omega\varepsilon_0 l} \frac{\left[\frac{\nu\omega_p^2}{\omega^3} \mathcal{I}_2 - \left(1 - \frac{\omega_p^2 - \nu^2}{\omega^2}\right)\mathcal{I}_1\right]}{\left[\left(1 - \frac{w_p^2}{\omega^2}\right)^2 + \frac{\nu^2}{\omega^2}\right]} \quad (19)$$

where $\mathcal{I}_1$ and $\mathcal{I}_2$ are the following numerical integrals:

$$\mathcal{I}_1 = \frac{2l^2}{a^2} \int_0^1$$
$$\cdot \frac{((1 + y^2 + \beta(1 - y^2))^2 - \alpha^2(1 - y^2)^2)}{[(1 + y^2 + \beta(1 - y^2))^2 + \alpha^2(1 - y^2)^2]^2} (1 - y^2) \, dy \quad (20)$$

$$\mathcal{I}_2 = \frac{2l^2}{a^2} \int_0^1$$
$$\cdot \frac{2\alpha(1 - y^2)^2[1 + y^2 + \beta(1 - y^2)]}{[(1 + y^2 + \beta(1 - y^2))^2 + \alpha^2(1 - y^2)^2]^2} \, dy. \quad (21)$$

These integrals can be derived in closed form for any given values of parameters α and β defined as

$$\alpha = \frac{l}{a} \text{Re}\left(\sqrt{-\frac{\varepsilon_1}{\varepsilon_3}}\right) \quad (22)$$

$$\beta = \frac{l}{a} \text{Im}\left(\sqrt{-\frac{\varepsilon_1}{\varepsilon_3}}\right). \quad (23)$$

C. Comparison

The resistance and reactance of the dipole under both current distributions are plotted in Fig. 3 versus frequency using a logarithmic scale along the ordinate displaying both negative and positive values of impedance.

We used plasma parameter values for a typical Earth's ionosphere at an altitude of 100 km. We see that the antenna impedances (both reactances and resistances) for both distributions are remarkably close. The frequencies of most interest for analyzing data lie in the regions where reactance becomes zero (known as the cyclotron frequency region and upper hybrid frequency region). The difference between reactances predicted by both theories is less than 1 percent in both regions.

We have not performed calculations for the more complicated case of an antenna with a 3-D exponential current distribution that is arbitrarily oriented with respect to the magnetic field. The current distribution components change with respect to the magnetic field but the distribution is still continuous and still decays with distance. Therefore, we expect that the differences between the impedances of this antenna and Balmain's will still be very small.

III. CONCLUSION

We showed that impedances of two antennas with very different current distributions (1-D and 3-D) are very close to each other. We conclude that the impedance of a dipole antenna immersed in a collisional plasma is insensitive to the specific form of current distribution assumed on the antenna. This is valid as long as an antenna is electrically short (quasi-static approach is valid) and the plasma is cold.

These results apply directly to the analysis of data from ionospheric impedance probes. The correct choice of a theory describing the antenna impedance is important. A knowledge of the complex antenna impedance versus frequency is needed. This allows to derive some of the plasma parameters by locating zero-crossing points of the dipole's reactance. We conclude that any of the two solutions for the antenna impedance considered in this paper can be used for such an analysis, as well as other possible solutions which satisfy the criteria discussed above.

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