Poynting Vector in Two Faces

In Section 4-8 of the textbook Pedrotti3, the magnetic flux density \( \mathbf{B} \) is used in the derivation of Poynting vector and subsequently the intensity, which is denoted as irradiance \( E_e \). Although in the book \( \mathbf{B} \) is also called magnetic field, but it is actually related to the magnetic field \( \mathbf{H} \) we use in our lecture notes by \( \mathbf{B} = \mu_0 \mathbf{H} \).

As you can noticed from Example 4-2 in the textbook, the unit for \( \mathbf{B} \) is T (Tesla) instead of A/m as used in our lecture notes.

For TEM waves, which Sec. 4-8 assumes, \( E_0 = \frac{\omega \mu_0 H_0}{k} = \frac{2 \pi f \mu_0 H_0}{2 \pi / \lambda} = c \mu_0 H_0 = c B_0 \).

This is then consistent with Eq. (4-30).

The following derivation will show you that Eqs. (4-40) and (4-42) in the textbook is the same as the equations we use to represent the Poynting vector and light intensity, which are \( \mathbf{S} = \mathbf{E} \times \mathbf{H} \) (assuming only real part of the fields are used as in the textbook) and \( I = \| \mathbf{E}_0 \|^2 / 2 \eta \).

Since \( \mathbf{B} = \mu_0 \mathbf{H} \) and \( c = 1/\sqrt{\mu_0 \varepsilon_0} \), Eq. (4-40) \( \mathbf{S} = \varepsilon_0 c^2 \mathbf{E} \times \mathbf{B} \) becomes \( \mathbf{S} = \mathbf{E} \times \mathbf{H} \).

Consequently, the first part of Eq. (4-42) becomes \( I = (E_0 \cdot H_0) / 2 = E_0^2 / 2 \eta \). The 2\(^{nd}\) and 3\(^{rd}\) parts of Eq. (4-42) then follows when you use the relations of \( \mathbf{B} = \mu_0 \mathbf{H} \) and \( c = 1/\sqrt{\mu_0 \varepsilon_0} \) again.

Note that in the text following Eq. (4-42), it says that when applying this equation to a medium of refractive index \( n \), \( \varepsilon_0 \) needs to be replaced by \( n^2 \varepsilon_0 \) and \( c \) need be replaced by \( c/n \). This is not obvious from the equation (a common mistake is to change \( c \) only but not \( \varepsilon_0 \)). Therefore, if possible, using the notations in our lecture notes will be less confusing.