EE485 Introduction to Photonics

Photon and Laser Basics

1. Photon properties
2. Laser basics
3. Characteristics of laser beams

Pattern formed by fluorescence of quantum dots

Reading: Pedrotti, Sec. 1.2, 6.4 - 6.8, 8.8, 27.1 - 27.5
Current flows for $\lambda < \lambda_0$, for any intensity of light

Apply stopping potential $V_0$

$\rightarrow$ Higher voltage required for shorter $\lambda$

The slopes are independent of light intensity, but current is proportional to light intensity.

\[ h\nu = eV_0 + w_0 \]

Energy of light = (Kinetic energy of the electron) + (Work function)

\[ E_{ph} = h\nu \]

Energy of quantum of light (Smallest energy unit): Photon

Energy of light = (Kinetic energy of the electron) + (Work function)
Photon Energy

\[ E = h \nu = \hbar \omega \]

\[ h = 6.63 \times 10^{-34} \text{ Joule} \cdot \text{Sec} \quad \text{: Plank’s constant} \]

\[ \hbar = h / 2\pi \]

**Figure 11.1-2** Relationships between photon frequency \( \nu \) (Hz), wavelength \( \lambda_o \), energy \( E \) (eV), and reciprocal wavelength \( 1/\lambda_o \) (cm\(^{-1}\)). A photon of wavelength 1 cm has reciprocal wavelength 1 cm\(^{-1}\). A photon of frequency \( \nu = 3 \times 10^{14} \) Hz has wavelength \( \lambda_o = 1 \mu\text{m} \), energy 1.24 eV, and reciprocal wavelength 10,000 cm\(^{-1}\).
Photon Momentum

Plane wave

\[ E(r, t) = A \exp(i k \cdot r - i \omega t) \hat{e} \]

Photon momentum

\[ p = \hbar k \]

\[ p = \hbar k = \frac{h}{\lambda} \]

Radiation Pressure
(assume the photons are absorbed)

\[ p = \frac{h}{\lambda} \]

Force:
\[ \frac{\Delta p}{\Delta t} \approx \frac{h/\lambda}{\Delta t} = N \frac{h}{\lambda} \]

Pressure:
\[ \frac{\text{Force}}{\text{Area}} \]

Exercise: An atom with mass \(10^{-25}\) kg emits a photon with energy 2.48 eV.
(1) What is the recoil velocity? (2) Compare this with the root-mean-square thermal velocity of the atom at \(T = 300K\).
Optical Tweezers

Forces arising from momentum change of the light

\[ F = \frac{\Delta P}{\Delta t} \]

\[ \Delta P = (c-d) \]

\[ \Delta P = (a-b) \]

Resultant “gradient” force

E.g., \( \lambda = 1064 \text{ nm}, P = 100 \text{ mW}, \) diameter of polystyrene sphere = 5 \( \mu \text{m} \to F = 3.18 \times 10^{-12} \text{ N.} \)

Photon Position

Heisenberg’s Uncertainty Principle

\[ \langle (\Delta z)^2 \rangle \langle (\Delta p)^2 \rangle \geq \frac{\hbar^2}{4} \]

Photon position cannot be determined exactly. It needs to be determined with probability.

**Probability** \( p(\mathbf{r})dA \propto I(\mathbf{r})dA \)

Example: (1) Photon position probability in a Gaussian beam.
(2) Transmission of a single photon through a beam-splitter.
Photon Polarization
— Linearly Polarized Photons

\[ E(\mathbf{r}, t) = (A_x \mathbf{\hat{x}} + A_y \mathbf{\hat{y}}) \exp[i(kz - \omega t)] \]
\[ E(\mathbf{r}, t) = (A_x' \mathbf{\hat{x}}' + A_y' \mathbf{\hat{y}}') \exp[i(kz - \omega t)] \]
\[ A_{x'} = \frac{1}{\sqrt{2}} (A_x - A_y), \quad A_{y'} = \frac{1}{\sqrt{2}} (A_x + A_y) \]

**Example:** Probability of observing a linearly polarized photon after a polarizer.

\[ p(\theta) = \cos^2 \theta \]
Quantum Communication

Secured information transmission with single photons

The Story of Alice, Bob and Eve
A Short Explanation of Polarized Photon Quantum Key Distribution

1. Alice generates single photons and passes them randomly through either a horizontal-vertical (+) or diagonal (X) polarizing filter.

2. She records the resulting polarization state of each photon—which she then correlates with either a "0" or a "1" to encrypt her message.

3. She sends the photons to Bob, who randomly chooses which filter should be used to measure the correct polarization state for each photon.

4. Bob tells Alice the filter sequence he used, and she tells him which parts of it he got right; that becomes the sifted encryption key.

5. If Eve, an eavesdropper, were to intercept the photons before they got to Bob, she would also have to guess the filter needed to record each polarization state. Since a wrong guess would change that state, Eve's presence will be detected when Alice and Bob compare notes.
Photon Polarization — Circularly Polarized Photons

\[
E(r, t) = (A_R \hat{e}_R + A_L \hat{e}_L) \exp[i(kz - \omega t)]
\]

\[
\hat{e}_L = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}) \quad \hat{e}_R = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y})
\]

Example: (1) A linearly-polarized photon transmitting through a circular polarizer.
(2) A right-circularly-polarized photon transmitting through a linear polarizer.
Single Photon Interference

Assume the mirrors and beam-splitters are perfectly flat and lossless. The beam-splitters are 50/50 beam-splitters. The path length difference is d.

Probability of finding the photon at the detector?

If we don’t find the photon, where is it?
Photon Flux Density and Photon Flux

Monochromatic light of frequency $\nu$ and intensity $I(r)$

Photon flux density

$$\varphi(r) = \frac{I(r)}{h\nu} \left( \frac{\text{photons}}{\text{sec} \cdot \text{area}} \right)$$

Photon flux

$$\Phi = \int_A \varphi(r) dA = \frac{P}{h\nu}, \quad P = \int_A I(r) dA : \text{Optical power (watts)}$$

Quasi-monochromatic light of central-frequency $\overline{\nu}$

Mean photon flux density

$$\varphi(r) = \frac{I(r)}{h\overline{\nu}}$$

Mean photon flux

$$\Phi = \frac{P}{h\overline{\nu}}$$

**TABLE 11.2-1 Mean Photon-Flux Density for Several Light Sources**

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Photon-Flux Density (photons/s-cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starlight</td>
<td>$10^6$</td>
</tr>
<tr>
<td>Moonlight</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Twilight</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>Indoor light</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Sunlight</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>Laser light (10-mW He–Ne laser beam at $\lambda_o = 633$ nm focused to a 20-μm-diameter spot)</td>
<td>$10^{22}$</td>
</tr>
</tbody>
</table>
Laser — Early History

*Laser: Light Amplification by the Stimulated Emission of Radiation*

Theoretical background: Stimulated emission, Einstein, 1916.

*Theodore H. Maiman and the first ruby laser.*
Einstein’s Theory of Light-Matter Interaction

\[ R_{St.Abs.} = B_{12} N_1 \rho(\nu) \]
\[ \rho(\nu) : \text{Spectral energy density} \]

\[ R_{Sp.Em.} = A_{21} N_2 \]
\[ N_2(t) = N_{20} e^{-A_{21}t} , \]
assume only spontaneous emission
\[ \tau = 1/A_{21} : \text{Spontaneous radiative lifetime} \]

Output photon: Same energy, direction, polarization and phase as the input photon.

\[ R_{St.Em.} = B_{21} N_2 \rho(\nu) \]
\[ A_{21} = \frac{8\pi h \nu_0^3}{c^3} \]
\[ B_{21} \]
\[ B_{12} = B_{21} \]
Essential Elements of a Laser

Pump: Creates population inversion

Laser medium: Provides amplifying medium

Resonator: Provide optical feedback and determine lasing wavelength
Laser Operation

**Lasing process in a ruby laser**

**Process:**
2. Seed photons — From spontaneous emission and initiate the stimulated emission process.
3. Optical cavity — Resonant enhancement and define the output wavelength.
   \[\rightarrow\] **Steady state.**
5. Output coupling — Let some of the photons out at each round trip.

Figure 21-7  Step-by-step development of laser oscillation in a typical laser cavity. (a) Quiescent laser. (b) Laser pumping. (c) Spontaneous and stimulated emission. (d) Light amplification begins. (e) Light amplification continues. (f) Established laser operation.
Fabry-Perot Cavity and Laser Resonators

Laser resonator

Fabric-Perot cavity

Laser amplifier
Exercise: A He-Ne laser has a cavity length of 60 cm. The gain medium is capable of supporting the laser light of wavelengths in the range from $\lambda_1 = 632.800 \text{ nm}$ to $\lambda_2 = 632.802 \text{ nm}$. (a) Determine the gain bandwidth in frequency. (b) How many longitudinal modes can exist in the laser cavity? (c) Find the maximum length $d$ of an etalon that could be used to limit this laser to single-mode operation.
Monochromaticity

Fluorescence (spontaneous emission)

Finite bandwidth at each energy level.

Lasing

Linewidth narrowed by stimulated emission and laser cavity

Lorentzian lineshape: \( g(\nu) = \frac{\Delta \nu_0}{2\pi[(\nu - \nu_0)^2 + (\Delta \nu_0/2)^2]} \)

<table>
<thead>
<tr>
<th>Light source</th>
<th>Center wavelength ( \lambda_0 ) (Å)</th>
<th>FWHM linewidth ( \Delta \lambda ) (Å)</th>
<th>FWHM linewidth ( \Delta \nu ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary discharge lamp</td>
<td>5896</td>
<td>≈ 1</td>
<td>9 \times 10^{10}</td>
</tr>
<tr>
<td>Cadmium low-pressure lamp</td>
<td>6438</td>
<td>≈ 0.013</td>
<td>9.4 \times 10^{8}</td>
</tr>
<tr>
<td>Helium-neon laser</td>
<td>6328</td>
<td>≈ 10^{-7}</td>
<td>7.5 \times 10^{3}</td>
</tr>
</tbody>
</table>
Directionality

**Divergence of a laser beam**

\[
\phi = \frac{1.27 \lambda}{D}
\]

**Fraunhofer diffraction of a plane wave through a circular aperture**

\[
\theta = \frac{2.44 \lambda}{D}
\]

Quiz: What makes the difference? (More on Gaussian beams later.)
Intensity of Light Sources

A 5-mW green laser of 532 nm wavelength emits from a 0.5 mm diameter aperture. Determine its intensity at 1 meter distance. How does this translate to number of photons per unit area per second?

(a) Find out the divergent angle.
(b) Determine the beam spot size at 1 meter distance.
(c) Calculate the intensity.
(d) Translate this into number of photons.

A 100-W light bulb has an average wavelength of 500 nm. Consider this as a point source, determine its intensity at 1 meter distance. How does this translate to number of photons per unit area per second?

(a) Find out the surface area at 1 meter distance.
(b) Calculate the intensity.
(c) Translate this into number of photons.
Focusability of Light

**Ideal situation**

Focusing an incoherent light

\[ \phi \approx 1.27 \frac{\lambda}{d} \]

Reversibility →

\[ d \approx 1.27 \frac{\lambda}{\phi} \geq \lambda, \text{ approximately} \]

Focusing a laser light

\[ \phi \approx 1.27 \frac{\lambda}{d} \]

Diffraction limit

Reversibility →

\[ d \approx 1.27 \frac{\lambda}{\phi} \geq \lambda, \text{ approximately} \]
Prelude to Gaussian Beam Optics

Review:

3-D wave equation
\[ \nabla^2 E - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \]

Plane wave solution
\[ \tilde{E}(r, t) = E_0 e^{i(k \cdot r - \omega t)} \]

Spherical wave solution
\[ \tilde{E}(r, \theta, \phi, t) = \frac{E_0 (\theta, \phi)}{r} e^{i(kr - \omega t)} \]

Paraxial approximation at far field:
At any point \((x, y)\) on the transverse plane \(z = R\),
\[ r = R \left( 1 + \frac{x^2 + y^2}{R^2} \right)^{1/2} \approx R + \frac{x^2 + y^2}{2R} \]
\[ \tilde{E}(x, y)_{z=R} = (\text{constant}) e^{ikr} \]
\[ \approx (\text{constant}) e^{ikR} e^{i(k/2)(x^2 + y^2)/R} \]
Laser-Beam Mode Structures

Measurement of a fundamental-mode laser beam:

← 3-D beam profile

Contour plot of intensity distribution →

Gaussian distribution:

$$I = I_0 e^{-2p^2/w^2}$$
Towards Gaussian Beam Solution

\[ E(r, t) = U(x, y, z)e^{i(kz - \omega t)} \]

Substitute into the wave equation,

\[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + 2ik \frac{\partial U}{\partial z} = 0 \]

Recall a far-field spherical wave under paraxial approximation,

\[ \tilde{E}(x, y)_{z=R} \cong (\text{constant})e^{ikR} e^{i(k/2)[(x^2 + y^2)/R]} \]

and take an “educated” guess:

\[ \tilde{U}(x, y, z) = E_0 e^{ip(z) + [k(x^2 + y^2)/2q(z)]} \]

\[ \frac{\partial p}{\partial z} = \frac{i}{q}, \quad \frac{\partial q}{\partial z} = 1 \]

Consider the Gaussian field distribution as well,

\[ E = E_0 e^{-(x^2 + y^2)/w^2} \]

Guess a complex radius of curvature \( q(z) \),

\[ \frac{1}{q(z)} = \frac{1}{R(z)} + i \frac{\lambda}{\pi w(z)^2} \]
$\tilde{E} = E_0 \frac{w_0}{w(z)} \exp \left[ -\frac{\rho^2}{w^2(z)} \right] \quad \text{Amplitude factor (Gaussian)}$

$\times \exp \left[ ik \frac{\rho^2}{2R(z)} \right] \quad \text{Radial phase}$

$\times \exp \left\{ i \left[ kz - \tan^{-1} \left( \frac{z}{z_0} \right) - \omega t \right] \right\} \quad \text{Longitudinal phase}$
Radius of Curvature, Spot Size, and Intensity Distribution

Define $z = 0$ as where the Gaussian beam wavefront is planar.

$$q(z) = q(0) + z$$

$$q(z) = z - i \frac{\pi w_0^2}{\lambda} \equiv z - iz_0$$

$$z_0 \equiv \frac{\pi w_0^2}{\lambda} : \text{Rayleigh range}$$

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_0} \right)^2}$$

$\rightarrow z = 0$ is where the beam waist is.

$$w(z_0) = \sqrt{2} w_0$$

$$I(\rho, z) = I_0 \left[ \frac{w_0}{w(z)} \right]^2 \exp \left[ - \frac{2\rho^2}{w^2(z)} \right]$$
Far Field and Divergence Angle

Far field region,

\[ z \gg z_0 \]

\[ R(z) = z \]

\[ w(z) = \frac{\lambda}{\pi w_0} z \]

Divergence angle

\[ \theta_{FF} = \frac{\lambda}{\pi w_0} \]
Exercise: Gaussian Beam in a Laser Cavity

He-Ne laser, $\lambda = 632.8$ nm

Determine:
(a) Beam radius $w_0$ at the beam waist.
(b) Rayleigh range $z_0$
(c) Beam radius $w$ at the rear laser mirror.
(d) Divergence angle.
Correspondence between $R(z)$ and $q(z)$

Extend the correspondence to simple lens law:

Ray-transfer matrix for laser propagation through arbitrary optical systems
Example 27-2: Determine the waist radius and its location for the He-Ne laser system depicted below.

(a) Determine $q_1$ at the flat mirror.
(b) Determine the ABCD matrix of the optical system.
(c) Relate $q_2$ at the beam waist to $q_1$. 

\[
R = \frac{y}{\alpha} \quad \begin{bmatrix} y_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix}
\]

\[
\rightarrow R_2 = \frac{AR_1 + B}{CR_1 + D}
\]

\[
q_2 = \frac{Aq_1 + B}{Cq_1 + D}
\]
Ray-transfer Matrix of Basic Optical Elements (I)

Translation matrix:

\[ M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \]

Refraction matrix, spherical interface:

\[ M = \begin{bmatrix} 1 & 0 \\ \frac{n - n'}{Rn'} & \frac{n}{n'} \end{bmatrix} \]

\( (+R): \text{convex} \)
\( (-R): \text{concave} \)

Refraction matrix, plane interface:

\[ M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{bmatrix} \]
Ray-transfer Matrix of Basic Optical Elements (II)

Thin-lens matrix:

\[ M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \]

\[ \frac{1}{f} = \frac{n' - n}{n} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Spherical mirror matrix:

\[ M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \]

\( (+f) : \text{convex} \)

\( (-f) : \text{concave} \)

\( (+R) : \text{convex} \)

\( (-R) : \text{concave} \)
Matrices of Cascaded Optical Components

\[
\begin{bmatrix}
    y_0 \\
    \theta_0
\end{bmatrix}
= M_1
\begin{bmatrix}
    y_1 \\
    \theta_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
    y_2 \\
    \theta_2
\end{bmatrix}
= M_2
\begin{bmatrix}
    y_1 \\
    \theta_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
    y_N \\
    \theta_N
\end{bmatrix}
= M
\begin{bmatrix}
    y_0 \\
    \theta_0
\end{bmatrix}
\]

Where

\[
M = M_N \cdots M_2 M_1 = \begin{bmatrix}
    A & B \\
    C & D
\end{bmatrix}
\]

Works particularly well with computer programming

A useful tip:

\[
\det M = AD - BC = \frac{n_o}{n_f}
\]
Gaussian Beam Through a Thin Lens

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = 
\begin{bmatrix}
1 & z' \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{bmatrix}
\begin{bmatrix}
1 & z \\
0 & 1
\end{bmatrix} = 
\begin{bmatrix}
1 - \frac{z'}{f} & z + z' - \frac{zz'}{f} \\
-\frac{1}{f} & 1 - \frac{z}{f}
\end{bmatrix}
\]

\[
q' = \left(1 - \frac{z'}{f}\right)q + \left(z + z' - \frac{zz'}{f}\right)
\]

Conditions:

\[
q = -i \frac{\pi (W_0)^2}{\lambda}, \quad q' = -i \frac{\pi (W_0')^2}{\lambda}
\]

Equate real parts and imaginary parts.
Focusing a Gaussian Beam

\[ z' = f + M^2(z - f) \]

For a plane wave, \( z' = f \)

For a Gaussian beam, \( z' \neq f \)

Beam waist at the lens \( z = 0 \)

\[ W_0' = \frac{W_0}{\sqrt{1 + \left(\frac{z_0}{f}\right)^2}} \]

\[ z' = \frac{f}{1 + \left(\frac{f}{z_0}\right)^2} \]

If \( z_0 \gg f \)

\[ W_0' = \frac{\lambda}{\pi W_0} f = \theta_0 f \]

\( z' = f \)

Assume \( W_0 \) is approximately equal to the radius of the lens,

\[ W_0' = \frac{2f\lambda}{\pi(2W_0)} = \frac{2\lambda(f\#)}{\pi} \]

\[ f\# = \frac{f}{D} : f\text{-number of the lens} \]