EE512A - Advanced Inference in Graphical Models — Fall Quarter, Lecture 1 http://j.ee.washington.edu/~bilmes/classes/ee512a_fall_2014/

Prof. Jeff Bilmes

University of Washington, Seattle Department of Electrical Engineering http://melodi.ee.washington.edu/~bilmes

Sep 29th, 2014



Prof. Jeff Bilmes

EE512a/Fall 2014/Graphical Models - Lecture 1 - Sep 29th, 2014

F1/34 (pg.1/106)

Announcements

• Welcome to the class!

• Mon, Wed, 11:30-1:30 in PCAR-297 (this room).

Logistics

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- Copies of (most) readings available on the web

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- Problem sets: answer a question, prove a theorem, etc.
- Alternatively programming projects, so you should be familiar with at least one programming language (e.g., C, C++, or matlab).
- All homework must be turned in electronically in PDF form via canvas at our assignment dropbox (https://canvas.uw.edu/courses/914697/assignments).

Final Project Possibility

Logistics

- There will be a final project, consisting of a 4-page conference-style paper, and a 10-minute final presentation.
- There will be a few milestones (1-page project proposal, 1-page progress summaries) during the quarter. These are graded.
- The final project should be regarding graphical models either as a user in an application, or as a researcher (i.e., new inference method, new proof, etc.).
- The date of the final project is tentatively Wednesday, December 10, 2014, 230-420 pm, PCAR 297.
- Final project reports due Tuesday, Dec 9th, at 11:45pm.
- All final project relate assignment must be turned in electronically via our class web page.

• There is a chance we will do a graphical model inference contest as the final project. More on this as the class progresses.

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- Lauritzen 1996 is a classic book on GMs.
- Two other books on Bayesian networks include Jensen 1996 and 2001.
- Two nice books on machine learning and pattern recognition are Bishop 2006 and Murphy 2012.

- Wainwright and Jordan *Graphical Models, Exponential Families, and Variational Inference* http://www.nowpublishers.com/product. aspx?product=MAL&doi=2200000001
- Markov Random Fields for Vision and Image Processing http://mitpress.mit.edu/catalog/item/default.asp?ttype= 2&tid=12668 edited by Andrew Blake, Pushmeet Kohli and Carsten Rother

- Reading assignment: Read the "trees.pdf" chapter soon to be posted on our canvas announcements page (https://canvas.uw.edu/courses/914697/announcements).
- Slides from previous time this course was offered are at our previous web page (http:

//j.ee.washington.edu/~bilmes/classes/ee512a_fall_2011/)
and even earlier at

http://melodi.ee.washington.edu/~bilmes/ee512fa09/.

Class Road Map - EE512a

- L1 (9/29): Introduction, Families, Semantics
- L2 (10/1):
- L3 (10/6):
- L4 (10/8):
- L5 (10/13):
- L6 (10/15):
- L7 (10/20):
- L8 (10/22):
- L9 (10/27):
- L10 (10/29):

- L11 (11/3):
- L12 (11/5):
- L13 (11/10):
- L14 (11/12):
- L15 (11/17):
- L16 (11/19):
- L17 (11/24):
- L18 (11/26):
- L19 (12/1):
- L20 (12/3):
- Final Presentations: (12/10):

Finals Week: Dec 8th-12th, 2014.

• This is where we will review previous lectures.

The Class	Inference and Learning	Graphical Models	GM Inference	Summary
Probabilis	tic Inference			

• Probabilistic Inference involves computing quantities of interest based on probability distributions. E.g., marginalization $\sum_{x_1} p(x_1, x_2)$ or maximization $\max_{x_1} p(x_1, \bar{x}_2)$.

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- Best of cases, exact inference is doable in polynomial time (trees).
- Worst of cases, exact inference is infeasible, so approximation is necessary
- Plethora of approximation methods are possible.
- The course this term will mostly concentrate on graphical model semantics, exact inference methods, and two broad approximation inference methods based on graphical models.


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- sum-product
- cluster variational methods
- expectation-propagation
- mean field methods
- max-product
- linear programming relaxations
- conic programming relaxations

and is therefore worthy of study. Of particular interest is for the class of exponential models (which have strong relationships to convexity).



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- Simplest of the ideas: use an efficient graph-cut approach to find the minimum energy configuration in a pairwise binary Markov random field.
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- Many generalizations, including move making algorithms such as alpha-beta swaps, alpha expansions, fusion moves, and other recent more sophisticated and energy aware "move making" algorithms.
- Computer vision and beyond.

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- Also, other modern search based methods.
- Beam pruning methods often go hand-in-hand with search based methods.

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Some no	tation			

Distributions

$$p(x) \equiv p(x_{1:N}) \equiv p(x_1, \dots, x_N) \equiv P_{X_1, \dots, X_N}(X_1 = x_1, \dots, X_N = x_N)$$

Subsets

$$V \stackrel{\Delta}{=} \{1, 2, \dots, N\} \quad A, B \subseteq V \quad A = \{a_1, \dots, a_{|A|}\}$$
(1.1)

$$X_{A} \stackrel{\Delta}{=} \{X_{a_{1}}, X_{a_{2}}, \dots, X_{a_{|A|}}\}$$
(1.2)

• Example: If $A=\{1,3,7\}$ then $X_A=\{X_1,X_3,X_7\}$ and

$$p(X_A = x_A | X_B = x_B) \equiv p(x_1, x_2 | x_3, x_4)$$

if $A = \{1, 2\}, B = \{3, 4\}$

• $p(x_A)$ requires table of size $r^{|A|}$, $r = |\mathsf{D}_X|$ where $\forall i, x_i \in \mathsf{D}_X$ • $\bar{x}^{(i)}$ and $\bar{x}^{(j)}$ are different vector samples for $i \neq j$.

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 - compute $p(\bar{x}_1, \bar{x}_2)$.
 - Given \bar{x}_1 compute

 $x_2^* \in \operatorname*{argmax}_{x_2} p(\bar{x}_1, x_2)$ or equivalently $x_2^* \in \operatorname*{argmax}_{x_2} p(x_2 | \bar{x}_1)$ (1.4)



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- Then find $\theta^* \in \operatorname{argmin}_{\theta} R(\mathbf{D}, \theta)$ where $R(\mathbf{D}, \theta)$ is a risk.
- Given θ^* , we may we interpret its values (generative learning)?
- Given θ^* , could form distribution $p_{\theta^*}(x)$ or marginal $p_{\theta^*}(x_1, x_2)$, etc.



• Generative learning if

$$R(\mathbf{D}, \theta) = -\frac{1}{|\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \log p_{\theta}(x^{(j)}) + \lambda \|\theta\|$$
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where $\|\cdot\|$ is some norm. This includes the case of maximum likelihood learning.



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• Discriminative learning results when

$$R(\mathbf{D}, \theta) = -\frac{1}{|\mathbf{D}|} \sum_{j=1}^{|\mathbf{D}|} \log p_{\theta}(x_2^{(j)} | x_1^{(j)}) + \lambda \|\theta\|$$
(1.6)

and includes the case of maximum conditional likelihood learning.



• Another form of discriminative learning, max-margin learning, occurs when if

$$R(\mathbf{D}, \theta) = \frac{1}{|\mathbf{D}|} \sum_{i=1}^{|\mathbf{D}|} \left[\max_{x_2} \left(\log p_{\theta}(x_2, x_1^{(j)}) + \Delta(x_2^{(j)}, x_2') \right) - \log p_{\theta}(x_2^{(j)}, x_1^{(j)}) \right] + \lambda \|\theta\| \quad (1.7)$$

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where $\Delta(x_2^{(j)},x_2')$ is a normalizing labeling cost. Overall, this corresponds to a generalized hinge-loss.

- Optimizing each risk is unique, but each invariably entails computing quantities over p(x) like the aforementioned inference.
- The need to efficiently compute with p(x) is critical for most machine learning problems.

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• Let ${\mathcal U}$ be the universe of all distributions over N r.v.s.



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- Let ${\mathcal U}$ be the universe of all distributions over N r.v.s.
- Sample data, along with domain knowledge, used to select resulting p(x) from \mathcal{U} that is "close enough" to $p_{\mathsf{true}}(x_1,\ldots,x_N)$.





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- Obtainable solution in *F* better than feasible solution in *U*



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- Size of U too large, complex, and with many local optima.
- Obtainable solution in *F* better than feasible solution in *U*
- Graphical models provide a framework for specifying $\mathcal{F} \subseteq \mathcal{U}$



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• A graphical model is a visual, abstract, and mathematically formal description of properties of families of probability distributions (densities, mass functions)

There are many types of graphical models, for example Markov random fields (left), factor graphs (center), and Bayesian networks (right):



The Class	Inference and Learning	Graphical Models	GM Inference	Summary
1111	11111			
Graphical	Models			

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The Class	Inference and Learning	Graphical Models	GM Inference	
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- Factorization requirements are often identical to conditional independence requirements.
- Factorization, in general, allows sums to be distributed into products thereby making (exact) inference quantities more efficient than if factorization properties did not exist.

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- A graphical model describes a family of distributions $p(x_V)$ over X_V .

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- In a GM, the constraints take the form of factorization (which are most often, conditional independence constraints).
- Factorization is useful since it allows for the distributive law to enable the use of dynamic programming for much faster exact inference than naive.
- Finding best way of doing inference is entirely graph theoretic operation.

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The Class	Inference and Learning	Graphical Models	GM Inference	Summary
1111	11111			
Graphie	cal Model			

- Each type of graphical model requires a certain type of graph (e.g., undirected, or DAG) and a set of rules (or "Markov properties") to define the GM.
- A graphical model is a pair $(G, \mathcal{M}) = ((V, E), \mathcal{M})$, a graph G and a set of properties \mathcal{M} that define what the graphical model means.
- <u>Conceptually</u>, one can think of a property $r \in \mathcal{M}$ is a predicate on a graph and a distribution, so $r(p, G) \in \{\text{true}, \text{false}\}$.
- (G, \mathcal{M}) consists of a family of distributions over x_V where all predicates hold. That is

$$\mathcal{F}(G, \mathcal{M}) = \{ p : p \text{ is a distribution over } X_V \text{ and }, \\ r(p, G) = \mathsf{true}, \forall r \in \mathcal{M} \}$$
(1.8)

• $\mathcal{F}(G, \mathcal{M}) \subseteq \mathcal{U}$

The Class	Inference and Learning	Graphical Models	GM Inference	Summary
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Marko	v Properties			

- Markov properties are rules that specify what are required of every family member. Any $p \in \mathcal{F}(G, \mathcal{M})$ satisfies all properties/rules $r \in \mathcal{M}$ for G. Any $p \in \mathcal{U} \setminus \mathcal{F}(G, \mathcal{M})$ violates at least one property for G.
- A $p \in U$ might have more properties. \mathcal{M} is like a filter, lets in those p that satisfy, but will let in those that satisfy more.



• Example $r \in \mathcal{M}$ might be "if there are two nodes $u, v \in V$ that are neither directly nor indirectly connected in G (i.e., there no path leading from u to v in G) then the corresponding random variables in p are independent"

The Class	Inference and Learning	Graphical Models	GM Inference	
Questions Needing to be	about Proper mathematically prove	rties		

- For a given type of graphical model, can the property set \mathcal{M} be listed in finite space and computed efficiently? (answer, yes).
- For a given type of graphical model, are there more than one set of rules that define a family? In other words, are there rule sets \mathcal{M}_1 and \mathcal{M}_2 such that $\mathcal{F}(G, \mathcal{M}_1) = \mathcal{F}(G, \mathcal{M}_2)$ for all G? Answer, yes.
- Much of the Lauritzen 1996 book studies graphs, rules (or Markov properties) and proves when the corresponding families are either identical, or subsets of each other.



• Is there a smallest rule set? In other words, are there rules sets $\mathcal{M}_1 \subset \mathcal{M}_2$ such that $\mathcal{F}(G, \mathcal{M}_1) = \mathcal{F}(G, \mathcal{M}_2)$, and can we compute the smallest set \mathcal{M}' so that $\mathcal{F}(G, \mathcal{M}') = \mathcal{F}(G, \mathcal{M})$ where $|\mathcal{M}'|$ is minimal?



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- Are there rule sets that are non-equivalent? I.e. \mathcal{M}_1 and \mathcal{M}_2 such that $\mathcal{F}(G, \mathcal{M}_1) \neq \mathcal{F}(G, \mathcal{M}_2)$ for some G? Answer, yes.



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- Are there rule sets that are non-equivalent? I.e. \mathcal{M}_1 and \mathcal{M}_2 such that $\mathcal{F}(G, \mathcal{M}_1) \neq \mathcal{F}(G, \mathcal{M}_2)$ for some G? Answer, yes.
- In general, much of graphical model theory is regarding deducing properties of rules and corresponding properties of graphs and the distributions they represent. This allows us to reason about graphs as a means of reasoning about families of distributions.



- $\mathcal{G}_N = \text{set of all undirected graphs over } N$ nodes.
- Consider

$$\mathcal{F}_N(\mathcal{M}) = \cup_{G \in \mathcal{G}_N} \mathcal{F}(G, \mathcal{M})$$
(1.9)

and

$$\mathcal{F}(\mathcal{M}) = \bigcup_{N=1}^{\infty} \mathcal{F}_N(\mathcal{M})$$
(1.10)

family of all distributions over any number of random variables that obeys rules ${\mathfrak M}$ for some undirected graph G.

- $\bullet \ \mathcal{M}$ determines the type of graphical model.
- $\mathfrak{M}^{(mrf)}$ rules for MRF, then $\mathcal{F}(\mathfrak{M}^{(mrf)})$ are the distributions representable by MRF.
- $\mathcal{M}^{(bn)}$ rules for Bayesian network, then $\mathcal{F}(\mathcal{M}^{(bn)})$ are the distributions representable by BN.

The Class	Inference and Learning	Graphical Models	GM Inference	Summary
Differen	t families			

- Families may be different.
- For a given graph G, we might have neither $\mathcal{F}(G, \mathcal{M}^{(\mathsf{mrf})}) \subset \mathcal{F}(G, \mathcal{M}^{(\mathsf{bn})})$ nor $\mathcal{F}(G, \mathcal{M}^{(\mathsf{bn})}) \subset \mathcal{F}(G, \mathcal{M}^{(\mathsf{mrf})})$.
- The relationship for the family in its entirety might be different. I.e., when we compare the set of all MRFs vs. the set of all BNs, i.e., $\mathcal{F}(\mathcal{M}^{(mrf)})$ vs. $\mathcal{F}(\mathcal{M}^{(bn)})$.
- Large part of GM research is understanding these relationships.

The Class	Inference and Learning	Graphical Models	GM Inference	
What is	graphical mode	el inference?		

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 - probability of evidence (marginalize the hidden variables)

 $p(\bar{x}_E)$

(1.11)



2 posterior probability, for $S \subseteq V \setminus E$ do

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 $\textcircled{O} most probable assignment, for <math>S \subseteq V \setminus E$ do

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- Given a graph G, we want to derive this just based just on (G, \mathcal{M}) and derive this automatically.
- We want to understand the computational complexity of the procedure based just on (G, \mathcal{M}) .
- amortization: we want to derive a procedure that works for any $p \in \mathcal{F}(G, \mathcal{M})$ for a given rule set.

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Sources	for Today's Leo	ture		

• Most of this material comes from the reading handouts that will soon be made available.