Advanced Introduction to Machine Learning

— Spring Quarter, Week 1 —

https://canvas.uw.edu/courses/1372141

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March 30th, 2020

Welcome!

- Welcome to the class: Advanced Introduction to Machine Learning
- An advanced introduction to machine learning and its applications in data science and artificial intelligence.

About the class name

Advanced Introduction to Machine Learning

- Introduction: assumes little background ML knowledge
- Advanced: requires good CS & Math and programming (mostly python, maybe C++) background. Also, there is a lot of material to cover!! Ambitious "(of a plan or piece of work) intended to satisfy high aspirations and therefore difficult to achieve."
- Machine Learning: what this class is about.

Logistics

- Mon/Wed 10:30-12:30 via zoom.
- Weekly virtual evening office hours: Thursdays, 10:00-11:00pm, via zoom (same link).
- our web page (`https://canvas.uw.edu/courses/1372141`), lecture slides to appear both before and after lecture.
- Use our discussion board (`https://canvas.uw.edu/courses/1372141/discussion_topics`) for all questions, comments, so that all will benefit from them being answered. Please use that rather then email.

Teaching Assistant

- Ricky Zhang <yz325@uw.edu>

Facts about the class

- Prerequisites: knowledge in probability, statistics, linear algebra, multivariate calculus, some information theory, mathematical optimization
- Text: We will be using class slides and drawing from material mostly available for free on the web although there are some recommended books as well (see other slide on references).
- Grades and Assignments: We will have approximately one assignment every two weeks. Each assignment is combination of math problems and large programming project(s) (in python). Grades are based on these assignments.
- No in-class tests, no final exam. Grade is based entirely on quality, clarity, and correctness in your HW problem sets and python ML projects.

More Facts about the class

- COLLABORATION POLICY: Homework must be done individually: each student must hand in their own answers. In addition, each student must write and submit their own code in the programming part of the assignment (we will run and test your code). It is acceptable, however, for students to collaborate in figuring out answers and helping each other solve the problems (but please use our assignment dropbox (`https://canvas.uw.edu/courses/1372141/assignments`)). You must also indicate on each homework with whom you collaborated.
- Homework assigned asynchronously via canvas, must be submitted electronically using our assignment dropbox (`https://canvas.uw.edu/courses/1372141/assignments`). Submissions: Single PDF file and, when relevant, .zip files with code/data/ipynb. Photos of very neatly hand written solutions, combined into one PDF, are fine
- Lecture slides will appear on canvas before the class begins, and updates post-lecture will also be posted with markups, as well as a youtube recording of each lecture.

Homework late policy

- For every additional day you take for the submission, we would subtract 5% of your grade.
- This continues for a week, after which we will stop accepting submissions and release the solutions.
- Example: $\text{EffectiveGrade} = \text{OriginalGrade} \times (1 - \text{LateDays} \times 0.05)$ where $0 \leq \text{LateDays} \leq 7$.
- If the homework is due on the 6th and you submit on the morning of 7th, you'll get max 95% of your grade, and so on up till 11:59 on the 13th – your last day to submit – when you will get max 65% of your grade.
- We recommend that you submit however much you have completed by the deadline.
- If you resubmit your assignment after the deadline, please make note of which problem(s) you have updated; we will only apply the reduction to that problem (entirely, not the subproblems).
- Also, note that we will not be giving any Late Days for the last homework, since there would be a very short window for us to grade it.

Religious Accommodations

Washington state law requires that UW develop a policy for accommodation of student absences or significant hardship due to reasons of faith or conscience, or for organized religious activities. The UW's policy, including more information about how to request an accommodation, is available at Religious Accommodations Policy (`https://registrar.washington.edu/staffandfaculty/religious-accommodations-policy/`). Accommodations must be requested within the first two weeks of this course using the Religious Accommodations Request form (`https://registrar.washington.edu/students/religious-accommodations-request/`).

Other logistics

- Almost all equations will have numbers.
- Equations will be numbered with lecture number, and number within lecture in the form $(\ell.j)$ where ℓ is the lecture number and j is the j^{th} equation in lecture ℓ. For example,

$$L(w) = \sum_{i=1}^{n} (y_i - x_i{}^{\mathsf{T}} w)^2 + \lambda \|w\|_2^2 \tag{1.1}$$

- Theorems, Lemmas, postulates, etc. will be numbered with $(\ell.s.j)$ where ℓ is the lecture number, s is the section number, and j is the order within that section.

Theorem 1.1.1 (foo's theorem)

foo

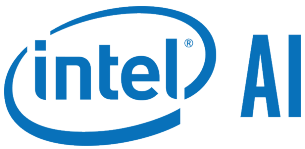
- Exception to these rules is in the review sections, where theorems, equation, etc. (even if repeated) will have new reference numbers.

Class Road Map

- W1(3/30,4/1): What is ML, Probability, Coins, Gaussians and linear regression, Associative Memories, Supervised Learning
- W2(4/6,4/8): More supervised, logistic regression, complexity and bias/variance tradeoff
- W3(4/13,4/15): Bias/Variance, Regularization, Ridge, CrossVal, Multiclass
- W4(4/20,4/22): Multiclass classification, ERM, Gen/Disc, Naïve Bayes
- W5(4/27,4/29): Lasso, Regularizers, Curse of Dimensionality
- W6(5/4,5/6): Curse of Dimensionality, Dimensionality Reduction, k-NN
- W7(5/11,5/13): k-NN, LSH, DTs, Bootstrap/Bagging, Boosting & Random Forests, GBDTs
- W8(5/18,5/20): Graphs; Graphical Models (Factorization, Inference, MRFs, BNs);
- W9(5/27,6/1): Learning Paradigms; Clustering; EM Algorithm;
- W10(6/3,6/8): Spectral Clustering, Graph SSL, Deep models, (SVMs, RL); The Future.

Last lecture is 6/8 since 5/25 is holiday (or we could just have lecture on 5/25).

Great Interest in Machine Learning/AI

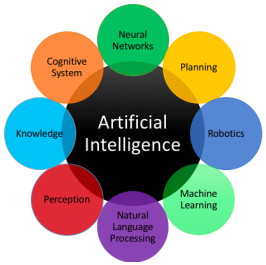


see more at https://nips.cc/Conferences/2018/Sponsors

Class Description

This is an ambitious class that will provide a broad overview of a large variety of machine learning methods in a short amount of time. You will learn to understand the basics of: linear regression; logistic regression; k-nearest neighbors; PCA, LDA, and dimensionality reduction methods; feature selection and engineering; cross validation; the bootstrap, bagging, and boosting; decision trees and random forests; naive Bayes; generative vs. discriminative models; support vector machines and kernel methods; neural networks; Bayesian nonparametric methods; clustering; ensemble methods; reinforcement learning; representation learning; information theory; Gaussian processes; supervised, unsupervised, and semi-supervised learning; graphical models; sparsity and compressed sensing; planning and control; information retrieval; structured prediction; matrix factorization; Monte Carlo methods; time-series analysis and HMMs; multi-agent learning; transfer and multi-task learning; active learning; submodularity; and machine teaching. Along the way, we will motivate the above using applications in computational biology, networks, computer vision, speech recognition, and natural language processing. We will also touch on the philosophy of machine learning and artificial intelligence, and discuss if we can build a computer program having artificial general intelligence. The class will require programming in python and the use of python libraries (e.g., numpy, sklearn, and pytorch). Previous knowledge of linear algebra, calculus, and basic probability theory and statistics is a must.

Class (and Machine Learning) overview

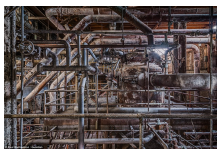


1. Introduction
- What is ML
- What is AI
- Why are we so interested in these topics right now?

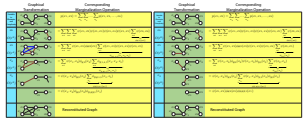
2. ML Paradigms/Concepts
- Overfitting/Underfitting, model complexity, bias/variance
- size of data, big data, sample complexity
- ERM, loss + regularization, loss functions, regularizers
- Supervised, unsupervised, and semi-supervised learning;
- reinforcement learning, RL, multi-agent, planning/control
- transfer and multi-task learning
- active learning
- machine teaching

3. Dealing with Features
- PCA, LDA, MDS, T-SNE, and dimensionality reduction methods
- Locality sensitive hashing (LSH)
- feature selection
- feature engineering
- matrix factorization & feature engineering
- representation learning

4. Evaluation
- accuracy/error, precision/recall, ROC, likelihood/posterior, cost/utility, margin
- train/eval/test data splits
- n-fold cross validation
- method of the bootstrap



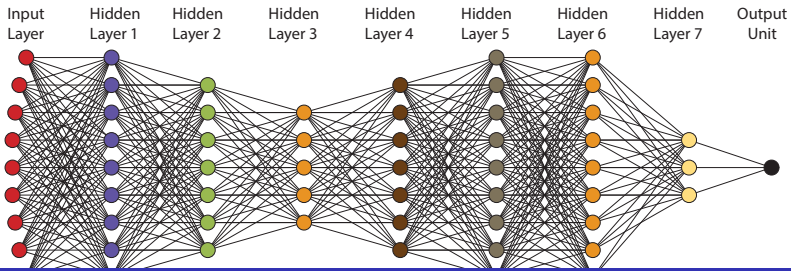
5. Optimization Methods
- Unconstrained Continuous Optimization: (stochastic) gradient descent (SGD), adaptive learning rates, conjugate gradient, 2nd order Newton
- Constrained Continuous Optimization : Frank-Wolf (conditional gradient descent), projected gradient, linear, quadratic, and convex programming
- Discrete optimization - greedy, beam search, branch-and-bound, submodular optimization.



6. Inference Methods
- probabilistic inference
- MLE, MAP
- belief propagation
- forward/backpropagation
- Monte Carlo methods

7. Models & Representation
- linear least squares, linear regression, logistic regression, sparsity, ridge, lasso
- generative vs. discriminative models
- Naive Bayes
- k-nearest neighbors
- clustering, k-means, k-mediods, EM & GMMs, single linkage
- decision trees and random forests
- support vector machines, kernel methods, max margin
- perceptron, neural networks, DNNs
- Gaussian processes
- Bayesian nonparametric methods
- ensemble methods
- the bootstrap, bagging, and boosting
- graphical models
- time-series, HMMs, DBNs, RNNs
- structured prediction
- grammars (as in NLP)

12. Other Techniques
- compressed sensing
- submodularity, diversity/homogeneity modeling



8. Philosophy, Humanity, Spirituality
- artificial intelligence (AI)
- artifical general intelligence (AGI)
- artificial intelligence vs. science fiction

9. Applications
- computational biology
- social networks
- computer vision
- speech recognition
- natural language processing
- information retrieval
- collaborative filtering/matrix factorization

10. Programming
- python
- libraries (e.g., NumPy, SciPy, matplotlib, scikit-learn (sklearn), pytorch, CNTK, Theano, tensorflow, keras, H2O, etc.
- HPC: C/C++, CUDA, vector processing

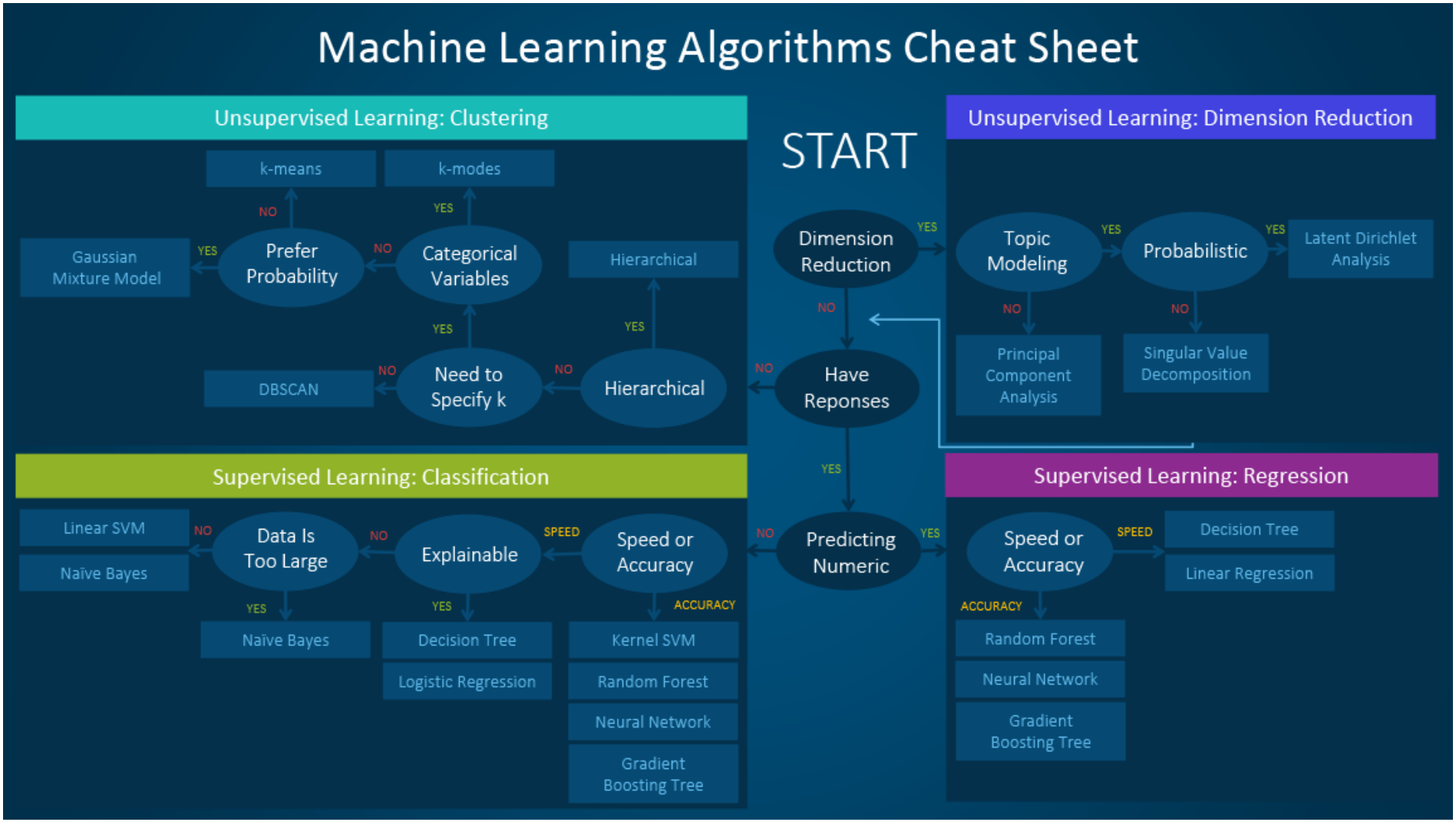
11. Background
- linear algebra
- multivariate calculus
- probability theory and statistics
- information theory
- mathematical (e.g., convex) optimization

1.2 Determinant

Let $\mathbf{A}$ be an $n \times n$ matrix.

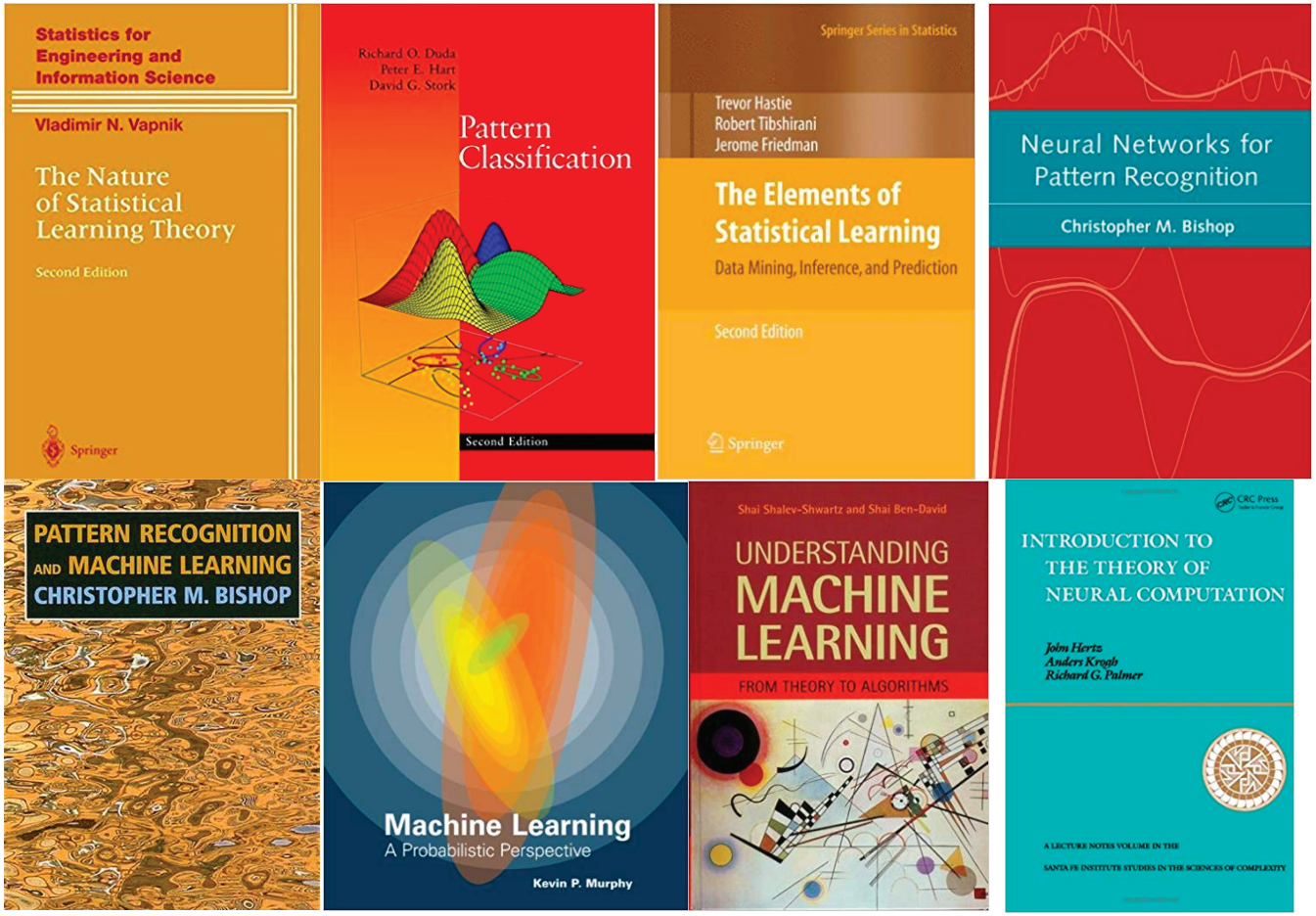
$$\begin{aligned}
\det(\mathbf{A}) &= \textstyle\prod_i \lambda_i \qquad \lambda_i = \mathrm{eig}(\mathbf{A}) \\
\det(c\mathbf{A}) &= c^n \det(\mathbf{A}), \qquad \text{if } \mathbf{A} \in \mathbb{R}^{n\times n} \\
\det(\mathbf{A}^T) &= \det(\mathbf{A}) \\
\det(\mathbf{AB}) &= \det(\mathbf{A})\det(\mathbf{B}) \\
\det(\mathbf{A}^{-1}) &= 1/\det(\mathbf{A}) \\
\det(\mathbf{A}^n) &= \det(\mathbf{A})^n \\
\det(\mathbf{I} + \mathbf{uv}^T) &= 1 + \mathbf{u}^T\mathbf{v}
\end{aligned}$$

Machine Learning (Grain of Salt) Cheat Sheet



from https://blogs.sas.com/content/subconsciousmusings/2017/04/12/machine-learning-algorithm-use/

Recommended References



- These class slides.
- Free resources

`https://towardsdatascience.com/list-of-free-must-read-machine-learning-books-89576749d2ff`

Machine Learning and Machine Intelligence

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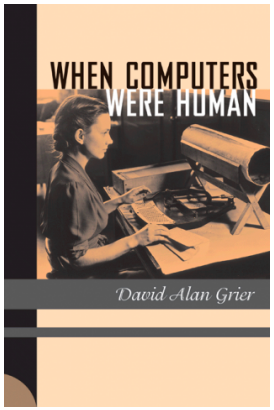
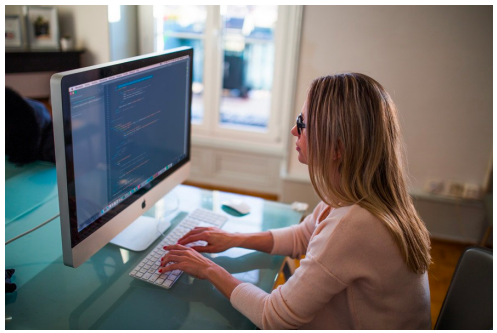
- IA: Indirect Algorithms
- Grand challenges: education, poverty, energy/climate change, and health.

Traditional Computer Programming vs. ML

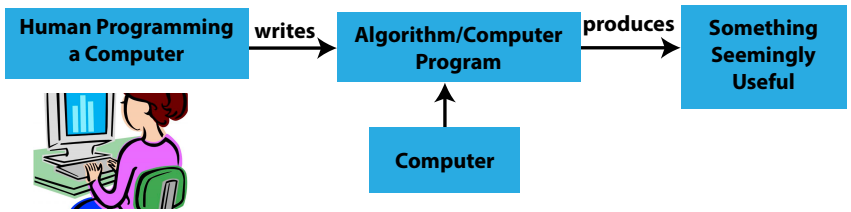


"It's a discipline note from my teacher. I solved the math problems in my head and I was supposed to use a calculator."



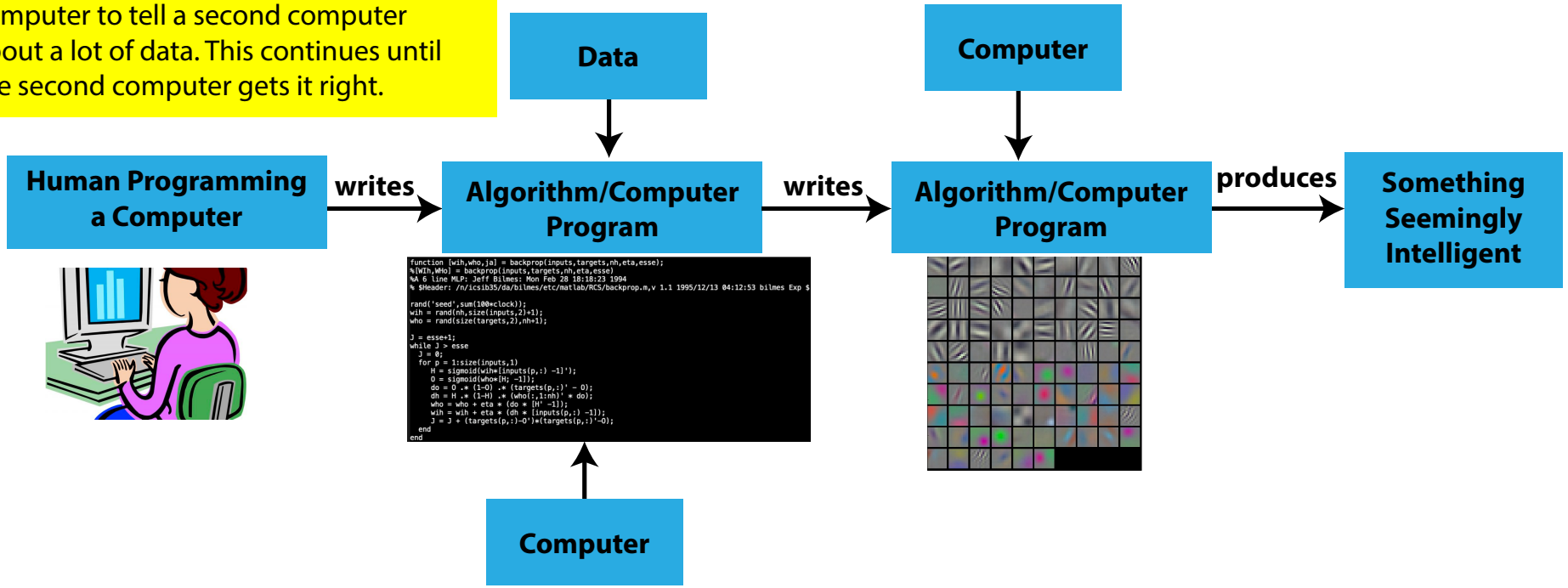
"Rather than learning how to solve that, shouldn't we be learning how to operate software that can solve that problem?"

Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a computer what to do, let us concentrate rather on explaining to human beings what we want a computer to do. -- **Donald Knuth**



Traditional Computer Programming vs. ML

Machine Learning is the art of repeatedly telling a computer what one wants the computer to tell a second computer about a lot of data. This continues until the second computer gets it right.



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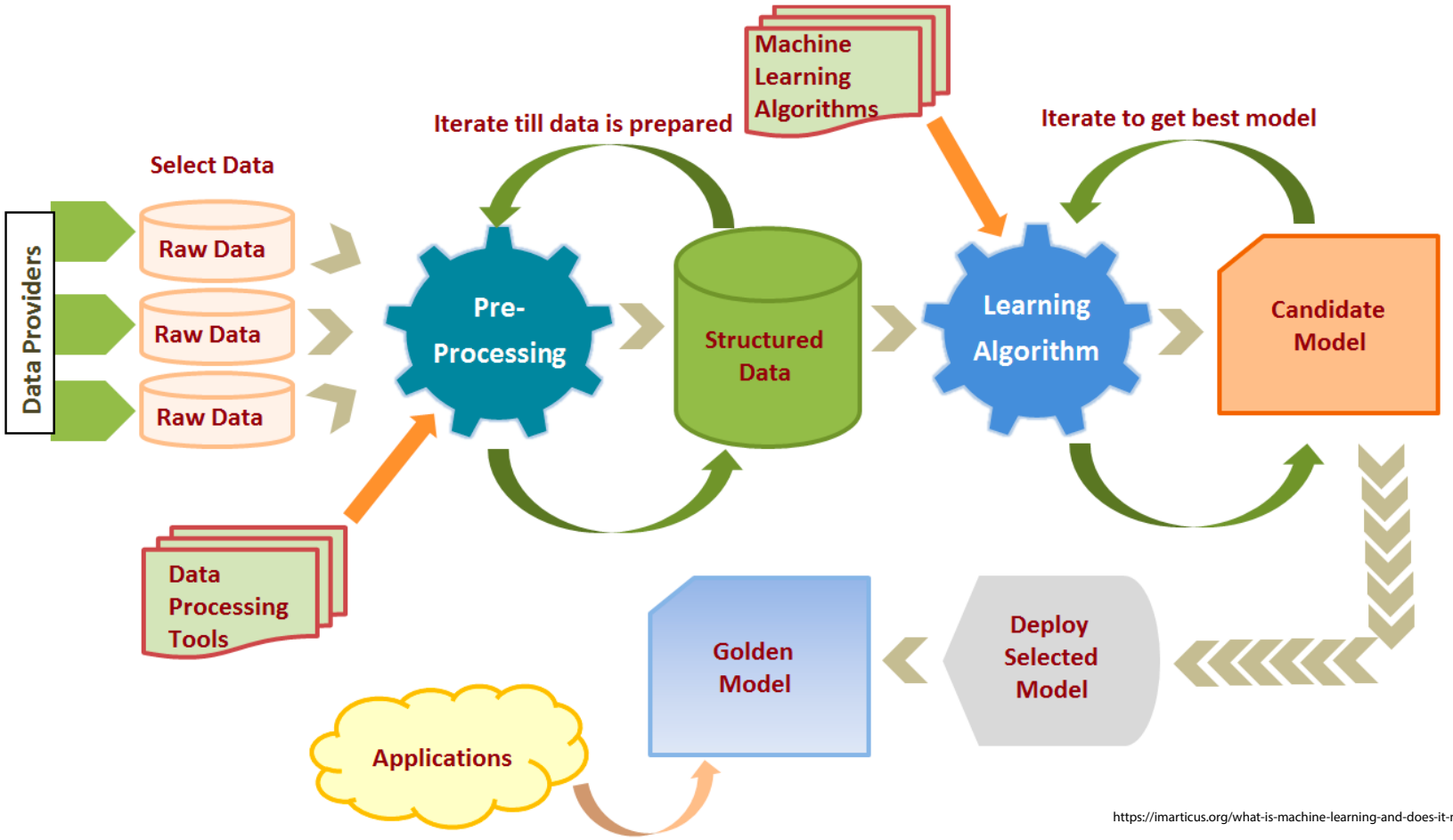
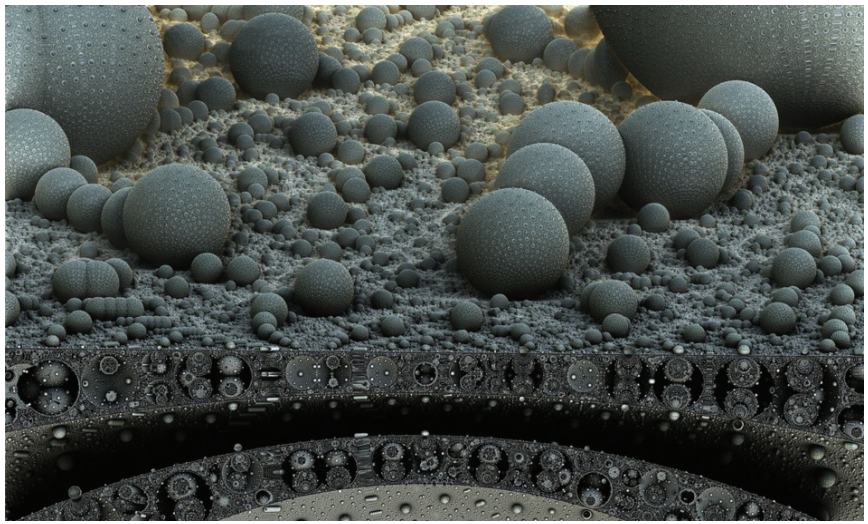


other defs of ML: `https://www.kdnuggets.com/2018/12/essence-machine-learning.html`

The Ideal Machine Learning Methods

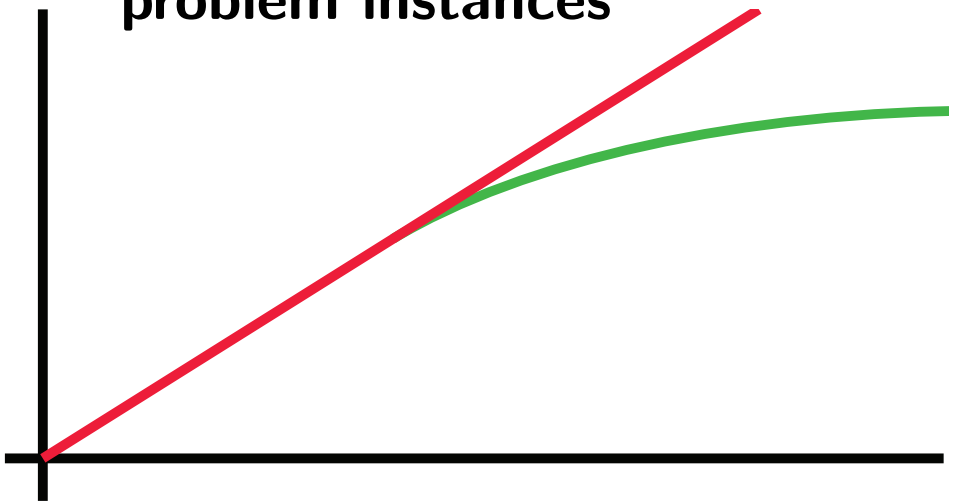
- **Simple to define**

- **Mathematically expressive**



- **Naturally suited to many real-world applications**

- **Efficient & scalable to large problem instances**



Object Recognition/Classification

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- And what are the following objects:






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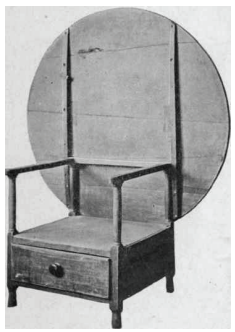
- Sometimes it is easy. Sometimes it is not so easy:


Object Recognition

- Sometimes it is a continuum (Escher, Liberation, 1955)

Object Recognition

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- What is foreground vs. background? (Escher, Mosaic, 1957)

Probability and Uncertainty

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- Machine learning often strives for the "best" probabilities in data using learning algorithms.

Coin Flipping and ML

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- How true, or likely, is it that θ is probability of heads?

$$\Pr(\mathcal{D}|\theta) = \theta^k(1-\theta)^{n-k} = \underline{\text{Likelihood}} \text{ of } \mathcal{D} \text{ given } \theta \tag{1.3}$$

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- How to find the most likely explanation of $\mathcal{D}$? Maximum likelihood

$$\hat{\theta}_{\mathsf{MLE}} = \operatorname*{argmax}_{\theta \in [0,1]} \Pr(\mathcal{D}|\theta) = \operatorname*{argmax}_{\theta \in [0,1]} \log \Pr(\mathcal{D}|\theta) \tag{1.4}$$

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- How to find $\hat{\theta}_{\mathsf{MLE}}$, calculus, $\frac{\partial}{\partial \theta} \log \Pr(\mathcal{D}|\theta) = 0$ leads to

$$\hat{\theta}_{\mathsf{MLE}} = k/n \tag{1.5}$$

Coin Flipping and ML

- $\mathcal{D} = \{b_1, b_2, \ldots, b_n\}$ is series of n independent and identical coin flips, $b_i \in \{\mathsf{H}, \mathsf{T}\}$.
- $k = |\{i : b_i = \mathsf{H}\}|$ is the count of the number of heads in $\mathcal{D}$
- How true, or likely, is it that θ is probability of heads?

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- Thus, computing k and dividing by n is a simple way to learn!

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- "There's no data like more data", and big data is pretty good, especially with lots of GPUs!
- In general, concentration inequalities (such as Markov's, Chebyshev's, Chernoff's, Hoeffding's, Bennet/Bernstein's, etc.) are useful to understanding properties of how quickly learning takes place. E.g., PAC learning is a form that allow us to compute the probability that a learnt model deviates from the true model.

Multivariate Gaussian (Normal), and samples thereof

- $\mathcal{D} = \{x_1, x_2, \ldots, x_n\}$ is series of n independent and identically distributed (i.i.d.) m-dimensional real-valued samples $\forall i, x_i \in \mathbb{R}^m$ from a Gaussian (or normal) distribution

$$x_i \sim \Pr(x|\mu, C) = \frac{1}{|2\pi C|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^\intercal C^{-1}(x-\mu)\right) \tag{1.7}$$

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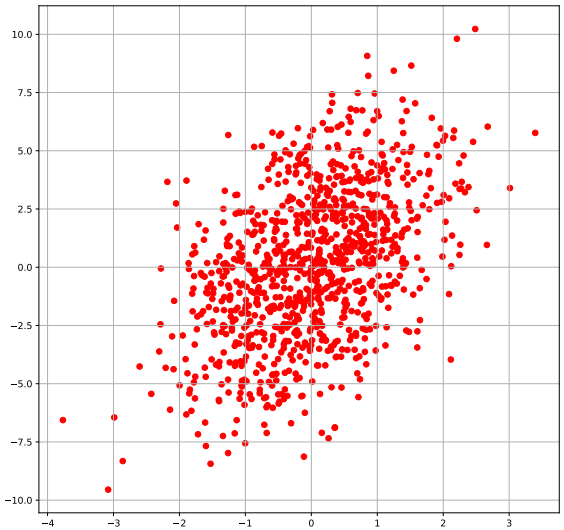
- Example sample of a 2D Gaussian.



Simple python code for sampling Gaussians

```
import numpy as np
import matplotlib.pyplot as plt

mean = [0,0]
cov = [[1, 1.5], [1.5, 10]]
X = np.random.multivariate_normal(mean, cov, 1000)

fig, ax = plt.subplots(figsize=(10, 10))
plt.scatter(X[:,0], X[:,1], c='r')
plt.grid()
plt.show()
fig.savefig("Gaussian_2D.pdf", bbox_inches='tight')
```

Useful environment for testing python: `https://jupyter.org`

Learning Gaussians

- Given the data sample $\mathcal{D}$ without knowing μ, C, how likely is the sample under some hypothesized parameters $\tilde{\mu}, \tilde{C}$.

$$\log \Pr(\mathcal{D}|\tilde{\mu}, \tilde{C}) = \sum_{i=1}^{n} \log \Pr(x_i|\tilde{\mu}, \tilde{C}) \tag{1.8}$$

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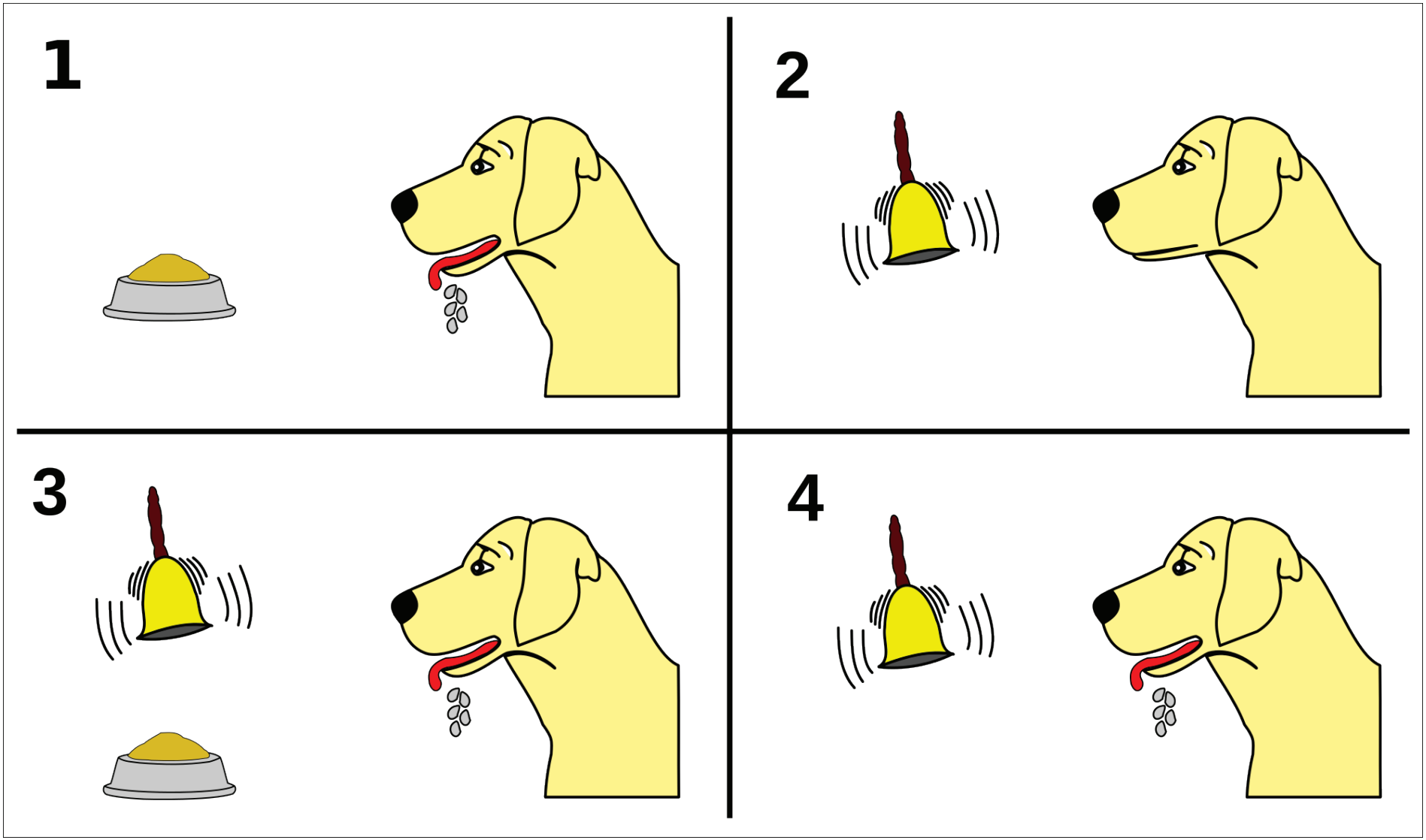
- How to find MLE quantities, again calculus, $\frac{\partial}{\partial \mu} \log \Pr(\mathcal{D}|\mu, C) = 0$ and $\frac{\partial}{\partial C} \log \Pr(\mathcal{D}|\mu, C) = 0$ leads to

$$\mu_{\mathsf{MLE}} = \frac{1}{n} \sum_{i=1}^{n} x_i \text{ and } C_{\mathsf{MLE}} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_{\mathsf{MLE}})(x_i - \mu_{\mathsf{MLE}})^{\mathsf{T}} \tag{1.11}$$

Associations and Associative Memories

Pavlov's Dog



Associations and Associative Memories

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- Why? $\mathcal{D}$ might not be complete, variation, noise, or possible data corruption not fully captured in $\mathcal{D}$. Also, $\mathcal{X}$ might be infinitely large.

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- Often, $x \in \mathbb{R}^m$ is an m-dimensional vector of features. In general, x is known as a feature vector.

1D Example

- Associating living area with price, or learn to map from living area to home price.

Living area (feet^2)	Price (1000$s)
2104	400
1600	330
2400	369
1416	232
3000	540
⋮	⋮

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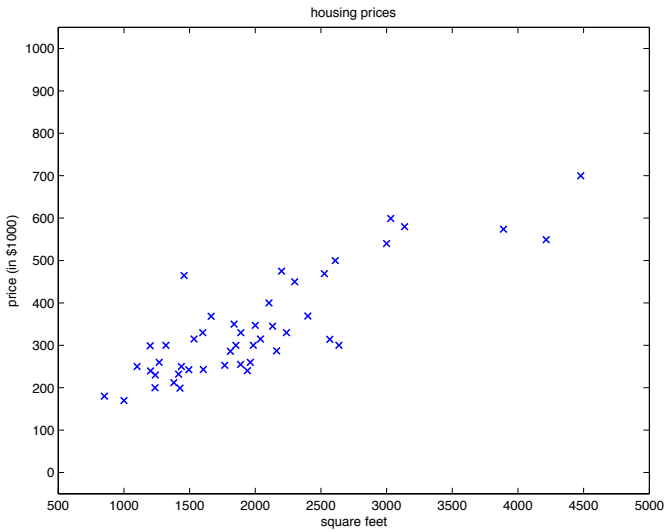

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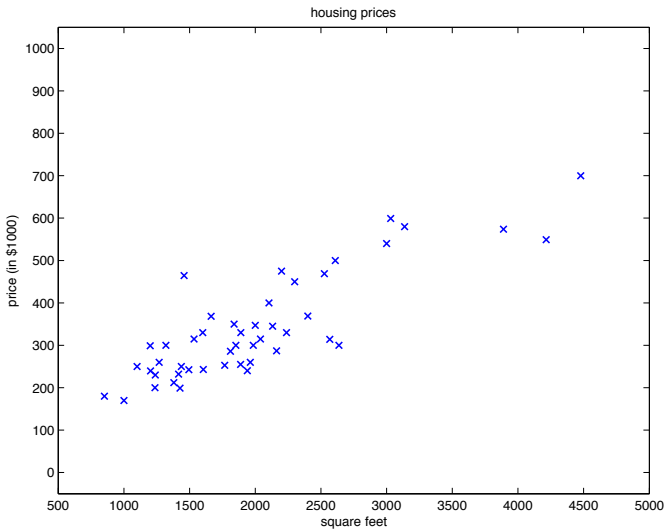

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- Here $\mathcal{X} = \mathcal{Y} = \mathbb{R}$.

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Higher Dimensions

- Can use other inputs as well.

Living area (feet^2)	#bedrooms	Price (1000$s)
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2400	3	369
1416	2	232
3000	4	540
⋮	⋮	⋮

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- Tradeoff: The good: more inputs, more information, more potential for accurate association. The bad: higher dimensional space, need for much larger n, curse of dimensionality.

Statistical parameter estimation

- Training data $\mathcal{D} = \left\{\left(x^{(i)}, y^{(i)}\right)\right\}_{i=1}^{n}$ where $\left(x^{(i)}, y^{(i)}\right) \sim p(x, y)$ are drawn from some distribution, $x^{(i)} \in \mathbb{R}^m$ and $y^{(i)} \in \mathbb{R}$.

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- This assumes we have the distribution p and also the resources to compute $E[Y|x]$.

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- We can find best solution under a linear model, where $h_\theta(x) = \theta^\mathsf{T} x$, where θ is an $m \times 1$ column vector of "regression" coefficients, in which case $\hat{y} = \theta^\mathsf{T} x$

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- Taking derivative of error objective $J(\theta)$ w.r.t. θ and set to zero gets:

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Linear estimator: Linear regression

- We can find best solution under a linear model, where $h_\theta(x) = \theta^\mathsf{T} x$, where θ is an $m \times 1$ column vector of "regression" coefficients, in which case $\hat{y} = \theta^\mathsf{T} x$
- In general, assume $h_\theta(x) \triangleq \theta^\mathsf{T} x$ is parameterized by parameters θ so

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- Linear $h_\theta(x) = x^\mathsf{T}\theta$ assumption, yields $\frac{\partial h_\theta(x^{(i)})}{\partial \theta} = x^{(i)}$.

Linear regression: feature vectors and transformations

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- Thus, while $h_\theta(x) = \theta^\intercal x = \theta^\intercal \phi(z)$ is linear in θ and x, it need not be linear (and is likely very non-linear) in z.
- Offers linear models much more expressiveness, ability, complexity (etc.), but also a risk for overfitting if too many features.

Linear Least Squares

- This gives objective to be minimized (smallest, or least of the sum of squares of the errors).

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{2}{n}\sum_{n=1}^{n}(x^{(i)\mathsf{T}}\theta - y^{(i)})x^{(i)} = 0 \tag{1.18}$$

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- We simplify this a bit by defining matrices associated with these quantities. First define a $n \times m$ design matrix X and length-n column vector $\vec{y}$

$$X = \begin{pmatrix} — & x^{(1)\mathsf{T}} & — \\ — & x^{(2)\mathsf{T}} & — \\ & \vdots & \\ — & x^{(n)\mathsf{T}} & — \end{pmatrix}, \text{ and } \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \tag{1.19}$$

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- Objective Equation (1.18), equivalent matrix-vector form:

$$J(\theta) = \frac{1}{2}(X\theta - \vec{y})^{\mathsf{T}}(X\theta - \vec{y}) \tag{1.20}$$

Normal Equations

- With this, we get the "normal equations"

$$\nabla_\theta J(\theta) = X^\intercal(X\theta - \vec{y}) = \vec{0} \tag{1.21}$$

i.e., modeling $\vec{y}$ to be in column space of matrix X (linear combinations of columns of X), when $\vec{y}$ is being approximated by $X\theta$.

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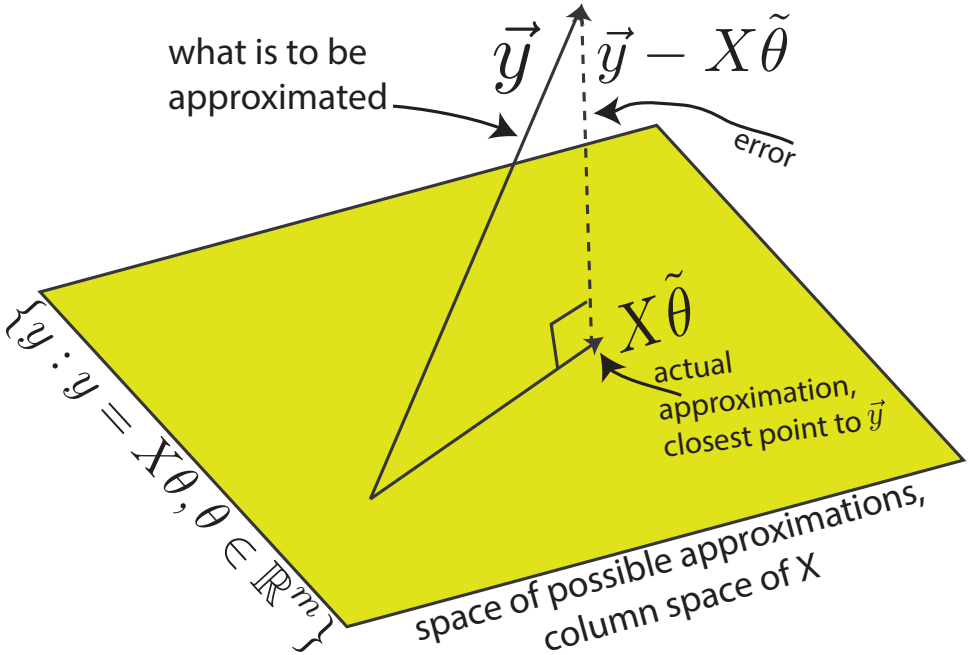

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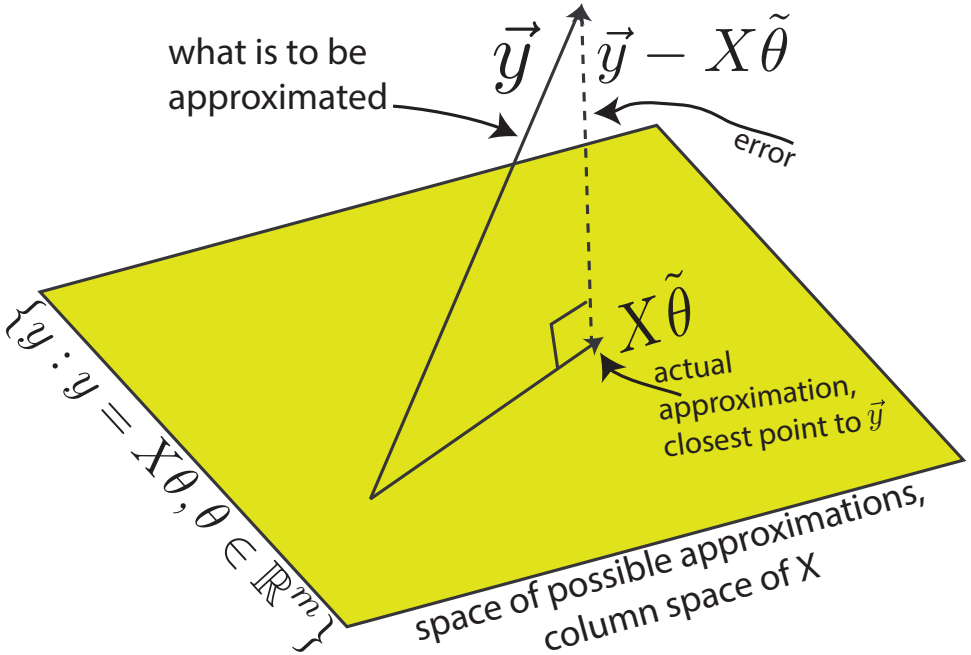

If $X^\intercal X$ invertible (typical if $n \gg m$), solution has form:

$$\tilde{\theta} = (X^\intercal X)^{-1} X^\intercal \vec{y}$$

where $(X^\intercal X)^{-1} X^\intercal$ is known as the Moore-Penrose pseudo-inverse of matrix X.

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- The approach taken is to move parameters θ in the direction of the gradient $\nabla_\theta J(\theta)$ a certain amount and to do that repeatedly.
- Gradient descent can itself both be batch and online.

Batch Gradient Descent

- Start with initial estimate $\theta \in \mathbb{R}^m$ and update it, for all $j \in [m]$ via

$$\theta_j \leftarrow \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta) \tag{1.22}$$

where

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{\partial}{\partial \theta_j} \frac{1}{2} (h_\theta(x) - y)^2 \tag{1.23}$$

$$= (h_\theta(x) - y) \frac{\partial}{\partial \theta_j} (h_\theta(x) - y) \tag{1.24}$$

$$= (h_\theta(x) - y) \frac{\partial}{\partial \theta_j} (\sum_j \theta_j x_j - y) \tag{1.25}$$

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- This leads to update rule, for all j:

$$\theta_j \leftarrow \theta_j + \alpha (y - h_\theta(x)) x_j \tag{1.27}$$

Batch Gradient Descent

- Gradient updates for all elements of θ at the same time and for sample pair $(x^{(i)}, y^{(i)})$

$$\theta \leftarrow \theta + \alpha(y^{(i)} - h_\theta(x^{(i)}))x^{(i)} = \theta + \alpha(y^{(i)} - \theta^\mathsf{T} x^{(i)})x^{(i)} \qquad (1.28)$$

  move θ in the direction of $x^{(i)}$ weighted by $\alpha(y^{(i)} - h_\theta(x^{(i)})) \in \mathbb{R}$, α times the error.

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- Batch Gradient Descent

Algorithm 3: Batch Gradient descent learning

Input : Training data $\mathcal{D}$, learning rate α, initial parameter estimate θ

Output: Learnt model parameters θ

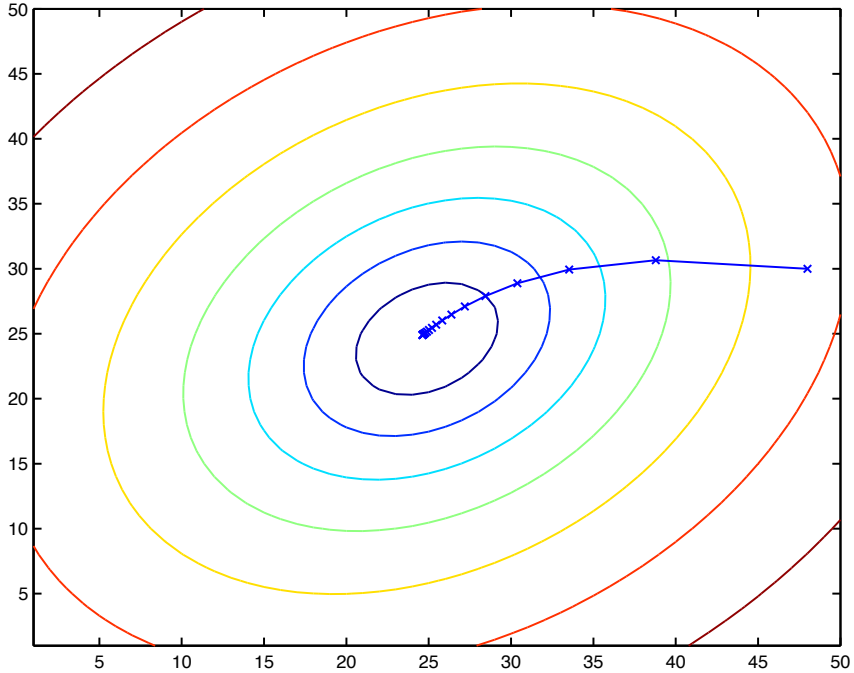
for $t = 1, \cdots, T$ **do**

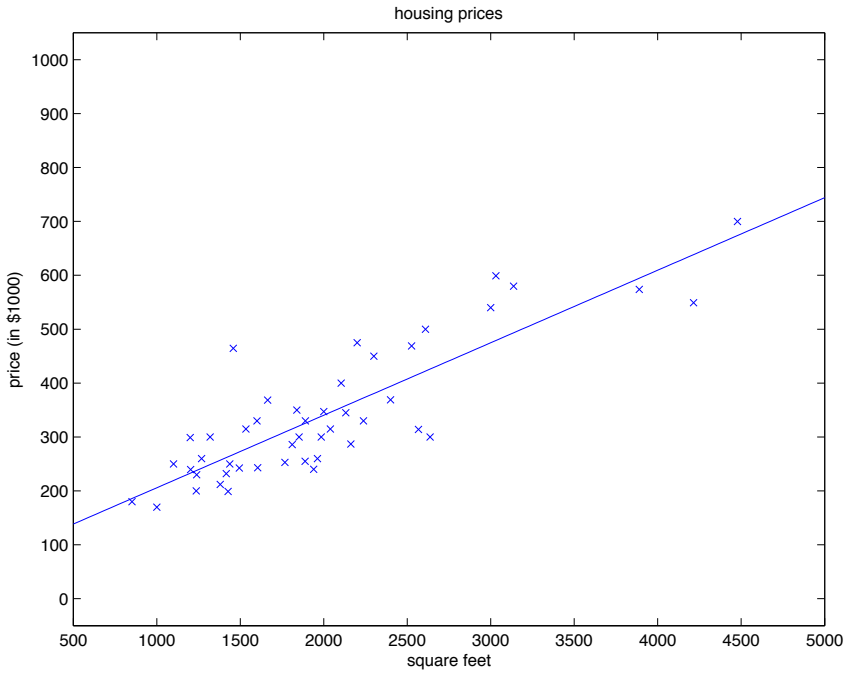
  $\theta \leftarrow \theta + \alpha \sum_{i=1}^{n} (y^{(i)} - h_\theta(x^{(i)}))x^{(i)}$

Return : the final parameters θ

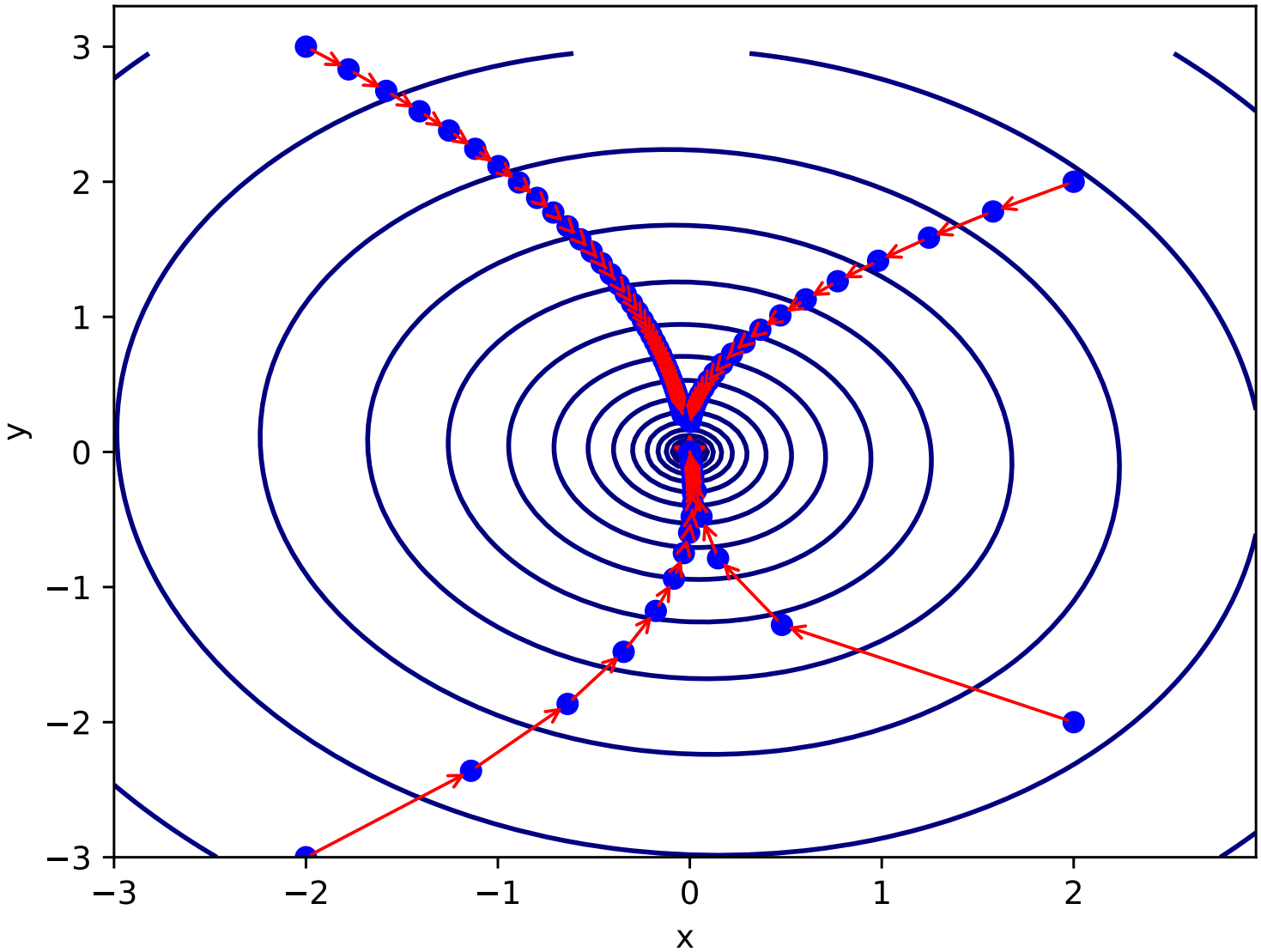
Batch Gradient Descent

Batch training results (left) and resulting fit model (right).





More visualization: Batch Gradient Descent



Stochastic Gradient Descent (SGD)

Algorithm 4: Stochastic gradient descent (SGD) learning

Input : Training data $\mathcal{D}$, learning rate α, initial parameter estimate θ

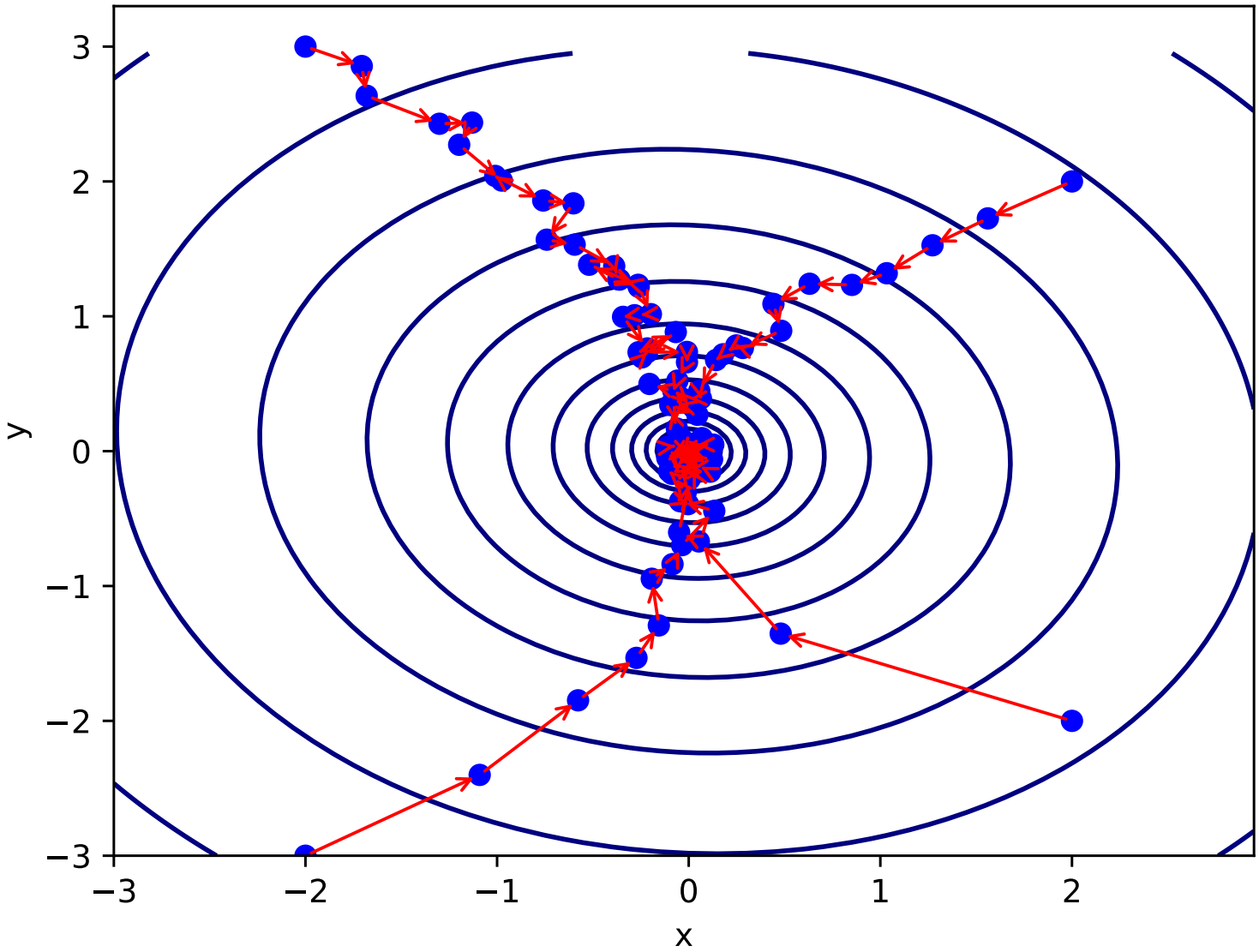
Output: Learnt model parameters θ

for $t = 1, \cdots, T$ **do**
  for $i = 1, \cdots, n$ **do**
    $\theta \leftarrow \theta + \alpha(y^{(i)} - h_\theta(x^{(i)}))x^{(i)}$

Return : the final parameters θ

Optimization folks (e.g., Bertsekas) call this incremental gradient methods. It is stochastic if we randomize (with or without replacement) the order of the data items.

More visualization: Stochastic Gradient Descent



Mini-Batch Gradient Descent

- Let $V = \{1, 2, \ldots, n\}$ be the index set of training points and let $\mathcal{V} = \{V_1, V_2, \ldots, V_k\}$ be a partition, $V_\ell \subseteq V$ and $V_\ell \cap V_p = \emptyset$ when $\ell \neq p$. Normally we want $|V_\ell| \approx |V_p|$ for all ℓ, p.

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- Each V_ℓ is a mini-batch (or bunch) of data points, leading to:

Algorithm 6: Minibatch stochastic gradient descent learning

Input : Training data $\mathcal{D}$, learning rate α, initial parameter estimate θ

Output: Learnt model parameters θ

for $t = 1, \cdots, T$ **do**
  for $\ell = 1, \cdots, k$ **do**
    $\theta \leftarrow \theta + \alpha \sum_{i \in V_\ell} (y^{(i)} - h_\theta(x^{(i)}))x^{(i)}$

Return : the final parameters θ

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Input : Training data $\mathcal{D}$, learning rate α, initial parameter estimate θ

Output: Learnt model parameters θ

for $t = 1, \cdots, T$ **do**
  for $\ell = 1, \cdots, k$ **do**
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Return : the final parameters θ

- Objective as sum of batch errors, of the form

$$J(\theta) = \frac{1}{k} \sum_{\ell=1}^{k} \frac{k}{n} |V_\ell| \left(\frac{1}{|V_\ell|} \sum_{i \in V_\ell} (h_\theta(x^{(i)}) - y^{(i)})^2 \right) \tag{1.29}$$

Normal Equations and Gaussians

- Assume $y^{(i)} = \theta^\intercal x^{(i)} + \epsilon^{(i)}$ where

$$\epsilon \sim \Pr(\epsilon^{(i)}) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(\epsilon^{(i)})^2}{2\sigma^2}) \tag{1.30}$$

or that

$$p(y^{(i)}|x^{(i)};\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(y^{(i)} - \theta^\intercal x^{(i)})^2}{2\sigma^2}) \tag{1.31}$$

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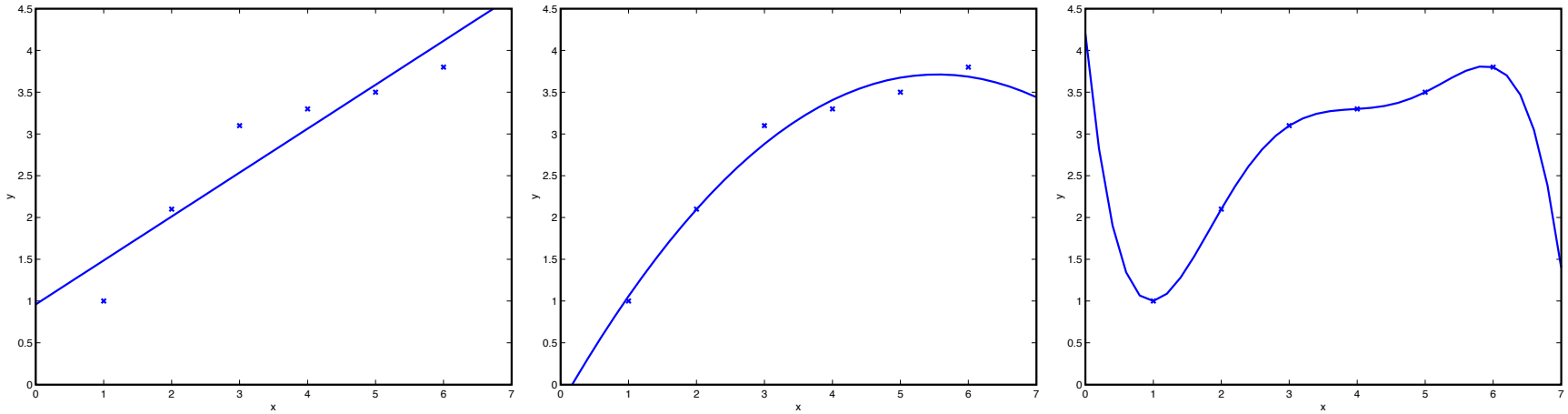
- Taking derivatives and setting them $= 0$ yields exactly same normal equations we saw earlier for solving linear least squares. Does not use σ.

Underfitting vs. Overfitting



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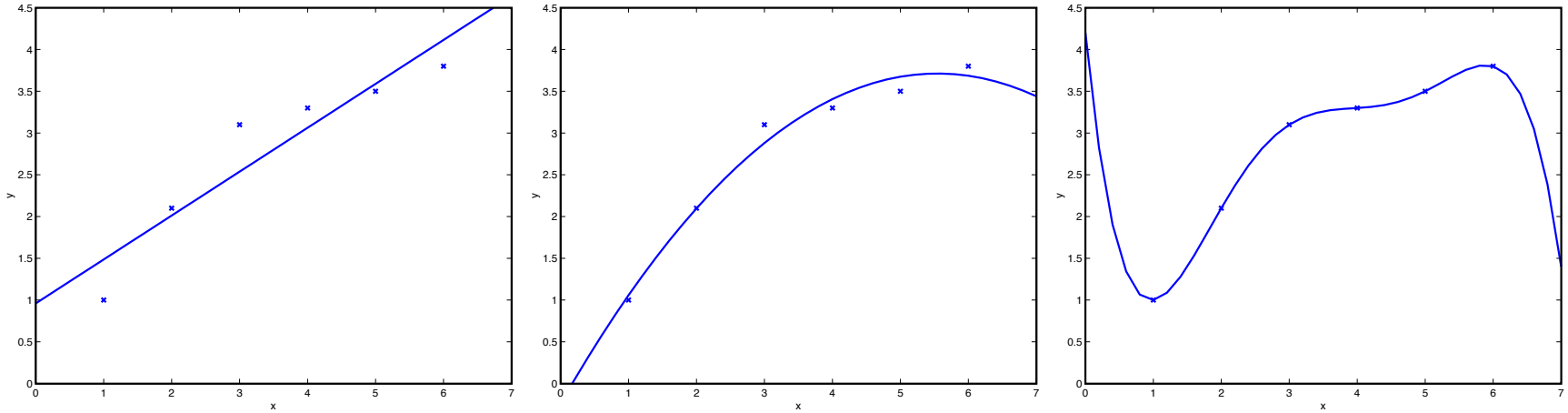

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- Fit models: left $y = \theta_0 + \theta_1 x$; middle $y = \theta_0 + \theta_1 x + \theta_2 x^2$; right $y = \sum_{j=0}^{5} \theta_j x^j$.

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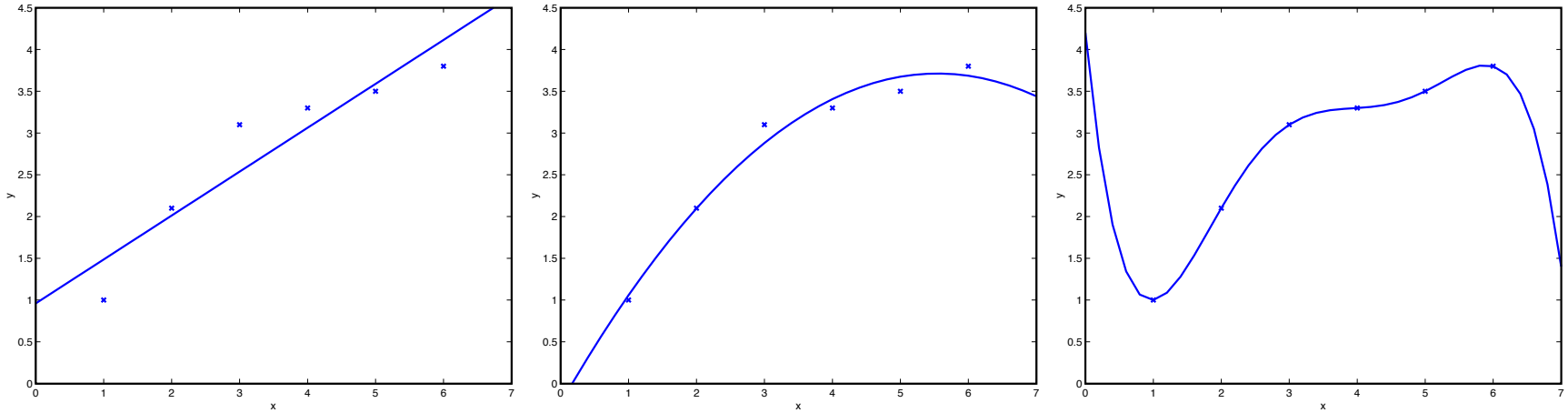

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- Left is underfitting. Right is overfitting.

Overfitting definition (T. Mitchell)

- We say that a hypothesis overfits the training examples if some other hypothesis that fits the training examples less well actually performs better over the entire distribution of instances (i.e., including instances beyond the training set).

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- We'll visit this topic again when we discuss bias/variance, but first lets discuss a few more models.